Robust Simultaneous Localization and Mapping for very large Outdoor Environments

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Abstract. This paper addresses the problem of Simultaneous Localization and Mapping (SLAM) when working in very large environments. A Hybrid architecture is presented that makes use of the Extended Kalman Filter to perform SLAM in a very efficient form and a Monte Carlo type filter to resolve the data association problem potentially present when returning to a known location after a large exploration task. The proposed algorithm incorporates significant integrity to the standard SLAM algorithms by providing the ability to handle multimodal distributions in real time. Experimental results in outdoor environments are presented to demonstrate the performance of the algorithm proposed.

1 Introduction

This paper address the problem of navigating autonomously in very large areas. This problem is usually referred as Simultaneous Localization and Mapping (SLAM) [1] or Concurrent Map Building and Localization (CML) [2]. It has been addressed in [3] using a Monte Carlo method in indoor problems and in [4] using sum of Gaussian in sub-sea applications. These algorithms are suitable to handle multi-modal probability distribution. Although these methods have proven to be very robust in many localization applications their extension to SLAM is computationally expensive making them difficult to apply in real time. In [5] the map building and localization processes are decoupled by assuming that the pose of the vehicle is known. This is achieved with enough particles to represent the true pose of the vehicle at all times. The Kalman Filter can also be extended to solve the SLAM problem [1] once appropriate models for the vehicle and sensors are obtained. In [6] the real time implementation aspects of SLAM using EKF techniques were addressed. A Compressed Extended Kalman Filter (CEKF) algorithm was introduced that significantly reduces the computational requirement without introducing any penalties in the accuracy of the results. Sub-optimal simplifications were also presented in [7] to update the full covariance matrix of the states bounding the computational cost and memory requirements.

Simultaneous navigation and map building algorithms are based on a exploration stage and re-visit of known places to register the new learned map to the known map. Depending on the quality of the kinematics models and external sensors used, the exploration stage can be extended to larger areas. Nevertheless no matter how good sensors and models are, at one point

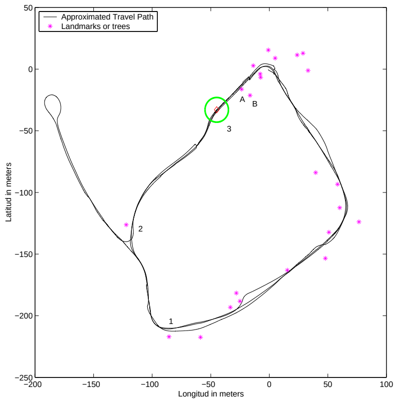

Fig. 1. Example of closing a large loop. The uncertainty when closing the loop at point 3 is larger than the separation between landmarks A and B.

the accumulated error will make the registration or data association task impossible. This problem is shown is Figure 1. In this experimental run the vehicle started near point 3 and circulated CW direction. The Figure shows the estimated path using a CEKF filter aided with absolute GPS information [8]. The " $*$ " represent natural landmarks incorporated as features into the map. The vehicle uses dead reckoning information to predict its position and incorporates features into the map to bound the dead-reckoning errors. If the vehicle return to the point label 3 with an error smaller than the separation between landmarks then it is possible to use standard algorithms to perform data association and register the new learned local map. In this particular case there are very few landmarks in the part of the trajectory labeled 1-2-3 making the estimated vehicle error to grow to 10 meters when returning close to the initial position labeled 3. This error is plotted as an ellipse in the Figure. Since the separation between the landmarks A and B is approximately 8 meters the algorithm will not be able to perform the data association and will be in failure. This is an inherent limitation of all simultaneous navigation and mapping methods and is independent of the implementation method or model used. Significant improvements to the standard Nearest Neighbor (NN) data association algorithm were presented in [9] with an approach based on considering more than one feature at a time. In this paper we present a robust solution to this problem using a combination of the CEKF with a Monte Carlo Filter. This hybrid architecture is designed to exploit the efficiency of the CEKF algorithm to estimate and maintain vehicle and map states and to provide appropriate initialization to the Monte Carlo filter to accelerate the convergence of the particle type filters once a potential data association problem is detected. The algorithms are presented for generic range/bearing, range only and bearing only sensors.

2 Bayesian Estimation in navigation problems

The SLAM problem under a probabilistic estimation approach requires that the marginal probability density $p(x_{L_k}, \mathbf{m}|\mathbf{Z}^k, \mathbf{U}^k, x_0)$ must be known for all k, where x_{L_k} is the vehicle state, x_0 its initial condition, $\mathbf{m}$ are the states representing a feature in the map and $\mathbf{Z}^k$ and $\mathbf{U}^k$ are the observations and input signals respectively at time k. To obtain the recursively form of this density [4] [10], it is assumed that the density $p(x_{L_{k-1}}, \mathbf{m}|\mathbf{Z}^{k-1}, \mathbf{U}^{k-1}, x_0)$ is known. Then applying the Bayesian rule and the Total Probability theorem we have

$$\begin{aligned} p(x_{L_k}, \mathbf{m}|\mathbf{Z}^k, \mathbf{U}^k, x_0) = \kappa p(z_k|x_{L_k}, \mathbf{m}) \cdot \\ \cdot \int p(x_{L_k}|x_{L_{k-1}}, u_k) p(x_{L_{k-1}}, \mathbf{m}|\mathbf{Z}^{k-1}, \mathbf{U}^{k-1}, x_0) dx_{L_{k-1}} \end{aligned} \tag{1}$$

where κ is a normalization constant, $p(z_k|x_{L_k}, \mathbf{m})$ represents the observation model and $p(x_{L_k}|x_{L_{k-1}}, u_k)$ models the vehicle dynamic. When the map $\mathbf{m}$ is known

$$\begin{aligned} p(x_{L_k}|\mathbf{m}, \mathbf{Z}^k, \mathbf{U}^k, x_0) = \kappa p(z_k|\mathbf{m}, x_{L_k}) \\ \int p(x_{L_k}|x_{L_{k-1}}, u_k) p(x_{L_{k-1}}|\mathbf{m}, \mathbf{Z}^{k-1}, \mathbf{U}^{k-1}, x_0) dx_{L_{k-1}} \end{aligned} \tag{2}$$

represents the *Localization Problem.*

2.1 Localization with the Particle Filter

Particle Filters approximate the joint posterior probability density with a set of random samples called particles. As the number of samples becomes large, they provide an exact, equivalent representation of the required distribution, that is the filter output will be close to the Bayesian filter. In this work we use the SIR (Sampling Importance Resampling) filter [11], to localize a vehicle in a predefined map using range and bearing information. Assuming that R samples $\{x^i_{k-1}\}^R_{i=1}$ of the previous posterior distribution are available, the process model is then used to propagate these samples to obtain $\{\tilde{x}^i_k\}^R_{i=1}$. The new samples represents the *a priori* probability density $p(x_k|\mathbf{m}, \mathbf{Z}^{k-1}, \mathbf{U}^k, x_0)$. The update stage is performed in two steps. The first step consist of the evaluation likelihood for each predicted particle as,

$$w_i = \frac{p(z_k|\mathbf{m}, \tilde{x}^i_k)}{\sum\limits_{j=1}^{R} p(z_k|\mathbf{m}, \tilde{x}^j_k)} \tag{3}$$

where z_k is the observation at time k. The pair $\{\tilde{x}^i_k\}^R_{i=1}$, $\{w^i_k\}^R_{i=1}$ defines a discrete distribution that tends to the real continuous *posterior* distribution as R tends to infinity. The second stage performs a resampling selecting only the particles with probability $p_r\{x^j_k = \tilde{x}^i_k\} = w^i_k$ for each j. Finally the probability of measuring z_k given the state $\tilde{x}_k$ is required, that is $p(z_k|\mathbf{m}, \tilde{x}^i_k)$. This pdf can be approximated with a sum of gaussian (SOG) assuming each beacon is represented with a Gaussian distribution centered at its estimated position and considering all the uncertainties presents in the observation:

$$p(z_k|\mathbf{m}, x_{L_k}) = \sum_{1}^{n} \frac{\alpha_i}{2\pi\sigma_r\sigma_\beta} e^{-0.5(\frac{(x_m - x_i)^2}{\sigma_x^2} + \frac{(y_m - y_i)^2}{\sigma_y^2})} \tag{4}$$

where (x_i, y_i) are the landmarks a priori estimated positions, σ_x and σ_y are the correspondent deviation in x and y and (x_m, y_m) are the observations obtained from each particle. For the range and bearing case they can be expressed as follows:

$$\begin{aligned} x_m &= \tilde{x_L}_k^i + z_r \cos(z_\beta - pi/2 + \tilde{\varphi_L}_k^i) \\ y_m &= \tilde{y_L}_k^i + z_r \sin(z_\beta - pi/2 + \tilde{\varphi_L}_k^i) \end{aligned} \tag{5}$$

where $(\tilde{x_L}_k^i, \tilde{y_L}_k^i, \tilde{\varphi_L}_k^i)$ are each of the states of the particles and (z_r, z_β) are the observations.

Localization with Range and Bearing Information In the case of range and bearing observation (z_r, z_β), it can be assumed that the measurements are contaminated by additive noise (γ_r, γ_β) with a given probability distribution. The conditional probability distribution of the observation (z_r, z_β) respect to the vehicle states, considering the uncertainty in the landmark position and the observation noise can be obtained from the following integral,

$$\begin{aligned} p(z_k|\mathbf{m}, x_{L_k}) &= \int_{\Omega} p(\mathbf{m}_x, \mathbf{m}_y, \gamma_r, \gamma_\beta)\ \mu\ |\overrightarrow{dS}| \\ \Omega &= \{(\mathbf{m}_x, \mathbf{m}_y, \gamma_r, \gamma_\beta) \in \Re^4\} \end{aligned} \tag{6}$$

The integral is a surface integral and $p(\mathbf{m}_x, \mathbf{m}_y, \gamma_r, \gamma_\beta)$ is the joint probability density distribution of the random variables (r.v.) due to the four noise sources. The factor $\mu \cdot |\overrightarrow{dS}|$ is the surface differential used to perform the integration over the surface region defined by the equality constrains given in equation 7.

$$\begin{aligned} z_r &= \sqrt{(\mathbf{m}_x - x_L)^2 + (\mathbf{m}_y - y_L)^2} + \gamma_r \\ z_\beta &= \arctan(\frac{\mathbf{m}_y - y_L}{\mathbf{m}_x - x_L}) - \varphi + \frac{\pi}{2} + \gamma_\beta \end{aligned} \tag{7}$$

The probability density distribution 6 can be evaluated using the probability density distribution restricted to the observations,

$$p_{z_r, z_\beta}(z_{r_0}, z_{\beta_0}) = \frac{\partial F_{z_r, z_\beta}(z_{r_0}, z_{\beta_0})}{\partial z_r \partial z_\beta} \tag{8}$$

where $F_{z_r,z_\beta}(z_{r_0}, z_{\beta_0})$ is the probability distribution. After some manipulations this distribution can be expressed as

$$F_{z_r,z_\beta}(z_{r_0}, z_{\beta_0}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\gamma r_0}\int_{-\infty}^{\gamma r_0} p(\mathbf{m}_x, \mathbf{m}_y, \gamma_r, \gamma_\beta)\cdot d\gamma_r \cdot d\gamma_\beta \cdot d\mathbf{m}_x \cdot d\mathbf{m}_y$$
$$\gamma_{r_0} = z_{r_0} - \sqrt{(m_x - x_L)^2 + (m_y - y_L)^2},$$
$$\gamma_{\beta_0} = z_{\beta_0} - \arctan(\tfrac{m_y - y_L}{m_x - x_L}) + \varphi - \tfrac{\pi}{2}, \quad (9)$$

Finally, the probability density distribution is evaluated differentiating the probability distribution function (9),

$$p_{z_r,z_\beta}(z_{r_0}, z_{\beta_0}) = \frac{\partial F_{z_r,z_\beta}(z_{r_0}, z_{\beta_0})}{\partial z_r \partial z_\beta} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p(\mathbf{m}_x, \mathbf{m}_y, \gamma_{r_0}(\mathbf{m}_x, \mathbf{m}_y), \gamma_{\beta_0}(\mathbf{m}_x, \mathbf{m}_y))d\mathbf{m}_x d\mathbf{m}_y \quad (10)$$

This integral is numerically evaluated reducing the integration region to the $\mathbf{m}_x, \mathbf{m}_y$ space close to the landmarks.

Localization with bearing only information In this case the observations model in 2-D is

$$z_{\beta_j} = \varphi_L - \arctan\frac{y_i - y_L}{x_i - x_L} + \frac{\pi}{2} \quad (11)$$

where (x_i, y_i) represent the position of the beacon i, (x_L, y_L, φ_L) is the vehicle position and β_j is the observed bearing to the beacon j. The probability $p(z_k|x_{L_k})$ can be calculated similarly to (eq. 10)

$$p_{z_\beta}(z_{\beta_0}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p(\mathbf{m}_x, \mathbf{m}_y, \gamma_{\beta_0}(\mathbf{m}_x, \mathbf{m}_y))\cdot d\mathbf{m}_x \cdot d\mathbf{m}_y \quad (12)$$

where

$$\gamma_{\beta_0} = \gamma_{\beta_0}(\mathbf{m}_x, \mathbf{m}_y) = \beta - \arctan\left(\frac{\mathbf{m}_y - y_L}{\mathbf{m}_x - x_L}\right) + \varphi_L - \frac{\pi}{2} \quad (13)$$

Localization with range only information Another common type of sensors are those that return range only information or range and bearing but with large uncertainty in bearing such as ultrasonic. The observations model can then be written

$$z_{r_j} = \sqrt{(y_i - y_L)^2 + (x_i - x_L)^2} \quad (14)$$

where (x_i, y_i) is the position of the observed beacon i, (x_L, y_L, φ_L) is the vehicle position and z_{r_j} is the j observation in the sensor data frame.

The conditional probability density distribution $p(z_k|x_{L_k})$ can be calculated according to (eq. 15) considering all the uncertainties present

$$p_{z_r}(z_{r_0}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p(\mathbf{m}_x, \mathbf{m}_y, \gamma_{r_0}(\mathbf{m}_x, \mathbf{m}_y)) \cdot d\mathbf{m}_x \cdot d\mathbf{m}_y \tag{15}$$

where

$$\gamma_{r_0} = z_{r_0} - \sqrt{(\mathbf{m}_x - x_L)^2 + (\mathbf{m}_y - y_L)^2} \tag{16}$$

3 Compressed filter and the aiding of the SIR filter

The proposed architecture uses CEKF under normal conditions to perform SLAM. At a certain time the system may not be able to perform the association task due to large errors in vehicle pose estimation. This is an indication that the filter can not continue working with a mono-modal probability density distribution. At this point, we have the CEKF estimated mean and deviation of the states representing the vehicle pose and landmark positions. With the actual map, a de-correlated map is built using a coordinate transform and the decorrelation procedures presented in [7]. A particle filter uses this information to resolve the position of the rover as a localization problem. When the multi-hypothesis are resolved the CEKF is restarted with the back propagated states values. Then the CEKF resumes operation until a new potential data association problem is detected. This section presents several important implementation issues that need to be taken into account to maximize the performance of the proposed architecture.

Map for the particle filter The SLAM algorithm builds a map while the vehicle explore a new area. The map states will be, in most cases, highly correlated in a local area. In order to use the particle filter to solve the localization problem a two dimensional map probability density distribution needs to be synthesized from an originally strongly correlated n dimension map. The decorrelation procedure is implemented in two steps. The map, originally represented in global coordinates is now represented in a local frame defined by two beacons states that are highly correlated to all the local landmarks. The other local landmarks are then referenced to this new base. This results in a covariance matrix of the form,

$$p\mathbf{m} = \begin{bmatrix} \mathrm{p}_{\mathbf{m}_1} & C_{12} & \cdots & C_{1m} \\ C_{21} & \mathrm{p}_{\mathbf{m}_2} & \cdots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ C_{m1} & \cdots & \ddots & \mathrm{p}_{\mathbf{m}_\mathrm{m}} \end{bmatrix} \tag{17}$$

where the cross-correlation components between states of different landmarks are usually weak, i.e. they meet the condition $C_{i,j}/\sqrt{p_{m_i} \cdot p_{m_j}} << 1$. To decorrelate the map it is necessary to apply an additional step. A conservative bound matrix for (eq. 17) can be easily obtained increasing the diagonal components and deleting the cross-correlation terms. This can be implemented as shown in eq 18 where $diag[\cdot]$ represents the elements of a diagonal matrix [7]. For presentation purposes, all the states in equation 18 are assumed to belong to different landmarks. The decorrelation procedure performs the decorrelation of block diagonal matrices, being each block matrix the covariance of the states representing a particular landmark.

$$\tilde{p}\mathbf{m} = diag\begin{bmatrix} p\mathbf{m}_1 + \sum_{j}^{j\neq 1} \mid k_{1j} \cdot C_{1j} \mid \\ \vdots \\ p\mathbf{m}_m + \sum_{j}^{j\neq m} \mid k_{mj} \cdot C_{mj} \mid \end{bmatrix} \tag{18}$$

The set $\{k_{ij}\}_{i,j}$ meets the condition $k_{ij} = 1/k_{ji} > 0$. This *un-correlated* map is used to define a two dimension map probability density distribution used by the particle filter to localize the vehicle.

Filter Initialization: As the number of particles affects both, the computational requirements and convergence of the algorithm, it is necessary to select an appropriate set of particles to represent the a priori density function at time T_0. Since the particle filters work with samples of a distribution rather than its analytic expression it is possible to select the samples based on the most probable initial pose of the rover. A good initial distribution is a set of particles that is dense in at least a small sub-region that contains the true states value. The initial distribution should be based in at least one observation in a sub-region that contains this true states value. Once a range and bearing from a landmark is obtained a distribution is created having a shape similar to a family of solid helical cylinders. Although it is recognized that some returns will not be due to landmarks, all range and bearing observations in a single scan are used to build the initial distribution. Even though a family of families of helices will introduce more particles than a single family of helices (one observation), it will be more robust in presence of spurious observations. Considering the observations of range and bearing as perfect observations, this defines a discontinued one dimensional curve (family of helices) C, in the three dimensional space (x, y, φ)

$$C_i = \left\{ (x, y, \varphi) \setminus \begin{array}{l} x = x(\tau) = x_i + z_r \cdot \cos(\tau) \\ y = y(\tau) = y_i + z_r \cdot \sin(\tau) \\ \varphi = \varphi(\tau) = \tau - z_\beta - \frac{\pi}{2} \\ \tau \in [0, 2\pi) \end{array} \right\} \tag{19}$$

These regions can be reduced by adjusting the variation of τ according to the uncertainty in φ. Assuming the presence of noise in the observation and in landmark position

$$\begin{aligned} z_r = z_r^* + \gamma_r, \;\; z_\beta = z_\beta^* + \gamma_\beta \\ x_i = x_i^* + \gamma_{x_i}, \;\; y_i = y_i^* + \gamma_{y_i} \end{aligned} \tag{20}$$

this family of helices becomes a family of cylindrical regions surrounding the helices.

Selection of a reduced local map: In most practical cases the local map is very large when compared to the sensor field of view. Most of the landmarks are usually beyond the range of the sensor. It is then possible to select only the *visible beacons* from the entire map taking into account the actual uncertainties. This will significantly reduce the computation complexity of (10). Figure 2 presents this approach for the case of only two particles. In this Figure (R, β) are the observations, the " $*$ " are the projected observation form each particles and the encircle stars are the beacons. It can be appreciated from the Figure that there are only a few beacons that can be within the field of view of any of the particles. The other beacons are not considered to be part of the reduced map.

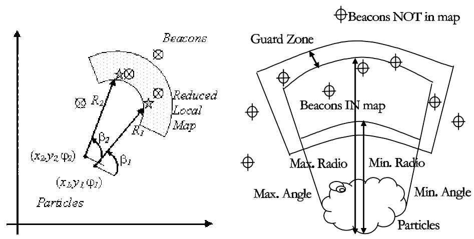

Fig. 2. Selected beacons in a reduced local map and uncertainty regions

Interface with the CEKF: Two main issues need to be addressed to implement the switching strategy between the CEKF and the SIR filter. The detection of the potential data association failure while running the CEKF is implemented by monitoring the estimated error in vehicle and local map states. The second issue is the reliable determination that the particle filter has resolved the multi-hypothesis problem and is ready to send the correct position to the CEKF back propagating its results. This problem is addressed by analyzing the evolution of the estimated deviations errors. The filter is assumed to converge when the estimated standard deviation error becomes less than two times the the noise in the propagation error model for x, y and φ.

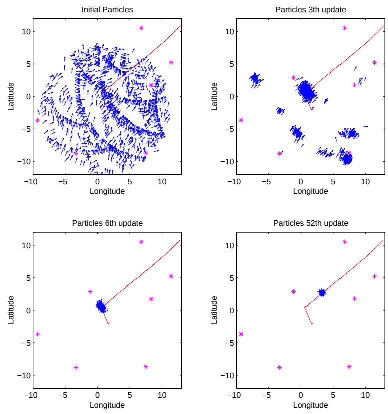

Fig. 3. GPS position and particles cloud after processing a number of observations: Top (left to right): Initialization and after 3 observations. Bottom (left to right): After 6 and 52 observations.

4 Experimental Results

This section presents experimental results of the proposed hybrid architecture running in an outdoor environment. The CEKF filter is used to navigate when no data association faults are detected otherwise the particle filter is initialized with the position and uncertainties reported by CEKF filter and is run after convergence is reached. This is shown in Figure 3 where the estimated path is represented at selected times for the range and bearing case. Figures 4 shows the deviations of the states x and y of the vehicle averaged over the fifty runs of the Monte Carlo simulation. For the range and bearing case, it is clear that convergence is achieved with the observations present in the first laser scan since the error is reduced during a single time stamp. The error at time 26 decreased from 2.2 to 0.5 meters. This scan included observations from several beacons. It is important to note that although the environment can be crowded with landmarks and other spurious objects the algorithm remains robust since no data association is performed at this stage. The convergence for the range only and bearing only case are much slower as expected but still achieved in few scans. Obviously, the convergence time will depend on the number of features in the environment.

5 Conclusions

This paper presented a hybrid architecture that make use of the Compressed Extended Kalman Filter (CEKF) algorithm to perform SLAM in an efficient form and a Monte Carlo type filter to resolve the data association problem present when closing large loops.

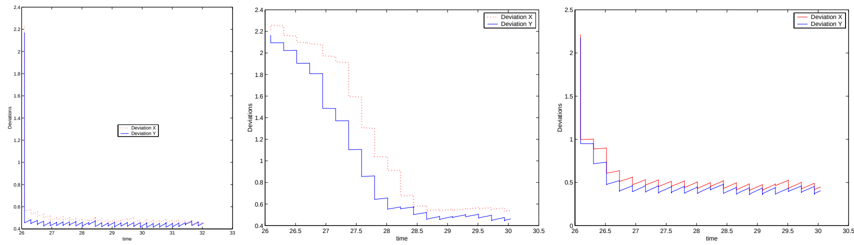

Fig. 4. History of state's x and y error when the filter is run with range and bearing, bearing only and range only information

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