

## MOSFET Small-Signal Model

- Concept: find an equivalent circuit which interrelates the incremental changes in  $i_D$ ,  $v_{GS}$ ,  $v_{DS}$ , etc. Since the changes are small, the small-signal equivalent circuit has linear elements only (e.g., capacitors, resistors, controlled sources)
- Derivation: consider for example the relationship of the increment in drain current due to an increment in gate-source voltage when the MOSFET is saturated-- with *all other voltages held constant*.

$$v_{GS} = V_{GS} + v_{gs}, i_D = I_D + i_d \text{ -- we want to find } i_d = (?) v_{gs}$$

We have the functional dependence of the total drain current in saturation:

$$i_D = \mu_n C_{ox} (W/2L) (v_{GS} - V_{Tn})^2 (1 + \lambda_n v_{DS}) = i_D(v_{GS}, v_{DS}, v_{BS})$$

Do a Taylor expansion around the DC operating point (also called the quiescent point or  $Q$  point) defined by the DC voltages  $Q(V_{GS}, V_{DS}, V_{BS})$ :

$$i_D = I_D + \left. \frac{\partial i_D}{\partial v_{GS}} \right|_Q (v_{gs}) + \frac{1}{2} \left. \frac{\partial^2 i_D}{\partial v_{GS}^2} \right|_Q (v_{gs})^2 + \dots$$

If the small-signal voltage is really “small,” then we can neglect all everything past the linear term --

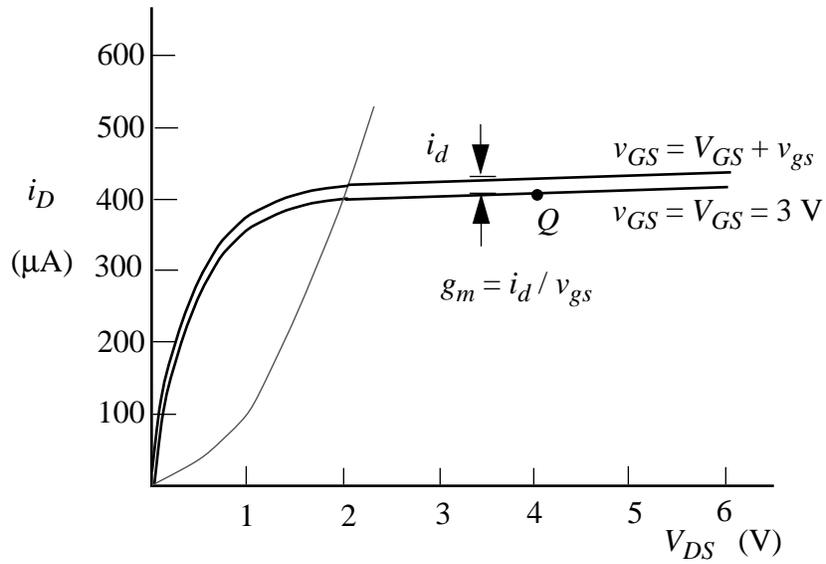
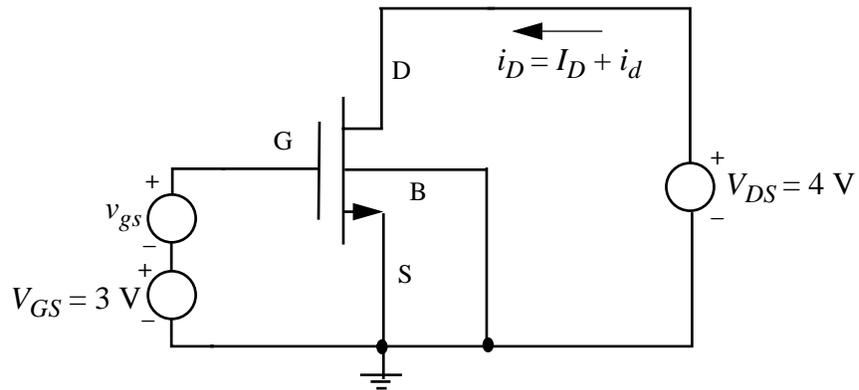
$$i_D = I_D + \left. \frac{\partial i_D}{\partial v_{GS}} \right|_Q (v_{gs}) = I_D + g_m v_{gs}$$

where the partial derivative is defined as the *transconductance*,  $g_m$ .

# Transconductance

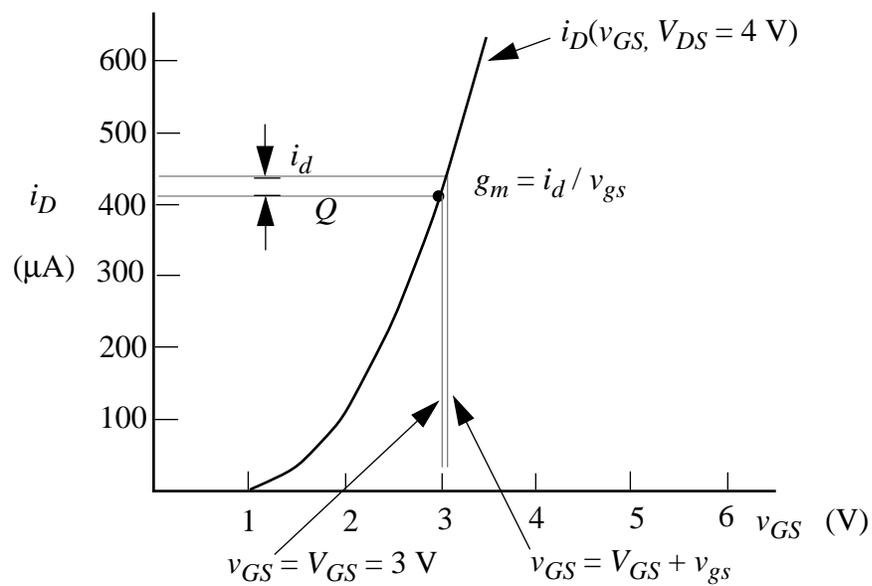
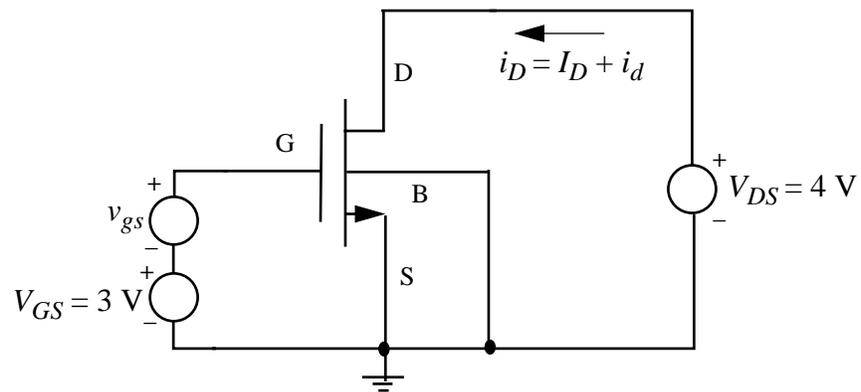
The small-signal drain current due to  $v_{gs}$  is therefore given by

$$i_d = g_m v_{gs}.$$



## Another View of $g_m$

\* Plot the drain current as a function of the gate-source voltage, so that the slope can be identified with the transconductance:



## Transconductance (cont.)

- Evaluating the partial derivative:

$$g_m = \mu_n C_{ox} \left( \frac{W}{L} \right) (V_{GS} - V_{Tn}) (1 + \lambda_n V_{DS})$$

Note that the transconductance is a function of the operating point, through its dependence on  $V_{GS}$  and  $V_{DS}$  -- and also the dependence of the threshold voltage on the backgate bias  $V_{BS}$ .

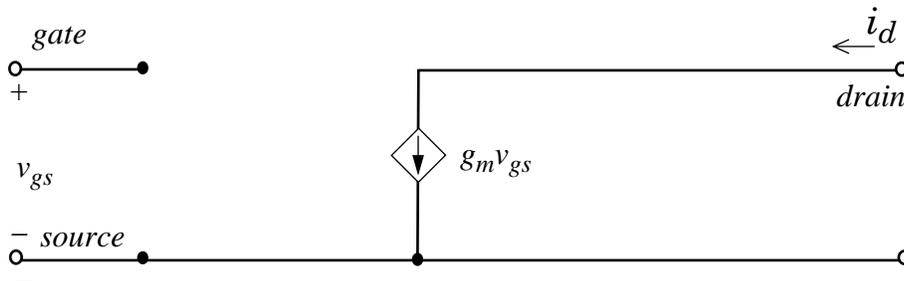
- In order to find a simple expression that highlights the dependence of  $g_m$  on the DC drain current, we neglect the (usually) small error in writing:

$$g_m = \sqrt{2\mu_n C_{ox} \left( \frac{W}{L} \right) I_D} = \frac{2I_D}{V_{GS} - V_{Tn}}$$

For typical values  $(W/L) = 10$ ,  $I_D = 100 \mu\text{A}$ , and  $\mu_n C_{ox} = 50 \mu\text{AV}^{-2}$  we find that

$$g_m = 320 \mu\text{AV}^{-1} = 0.32 \text{ mS}$$

- How do we make a circuit which expresses  $i_d = g_m v_{gs}$ ? Since the current is not across the controlling voltage, we need a voltage-controlled current source:



# Output Conductance

- We can also find the change in drain current due to an increment in the drain-source voltage:

$$g_o = \left. \frac{\partial i_D}{\partial v_{DS}} \right|_Q = \mu_n C_{ox} \left( \frac{W}{2L} \right) (V_{GS} - V_T)^2 \lambda_n \cong \lambda_n I_D$$

The output resistance is the inverse of the output conductance

$$r_o = \frac{1}{g_o} = \frac{1}{\lambda_n I_D}$$

The small-signal circuit model with  $r_o$  added looks like:

