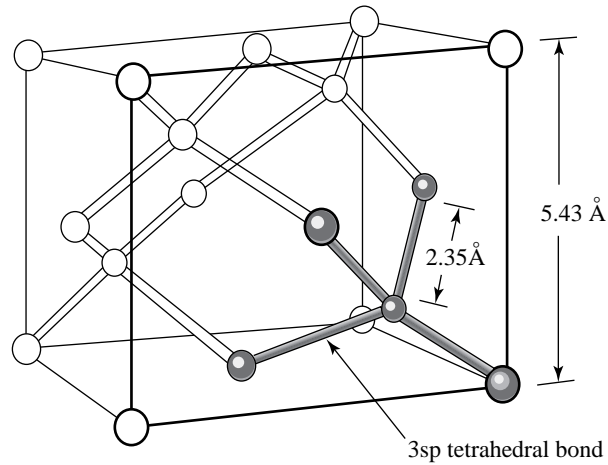
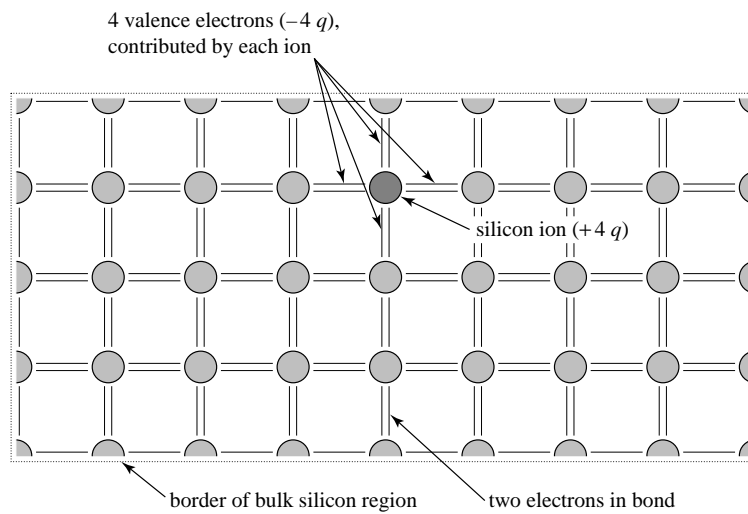


# Silicon Crystal Structure

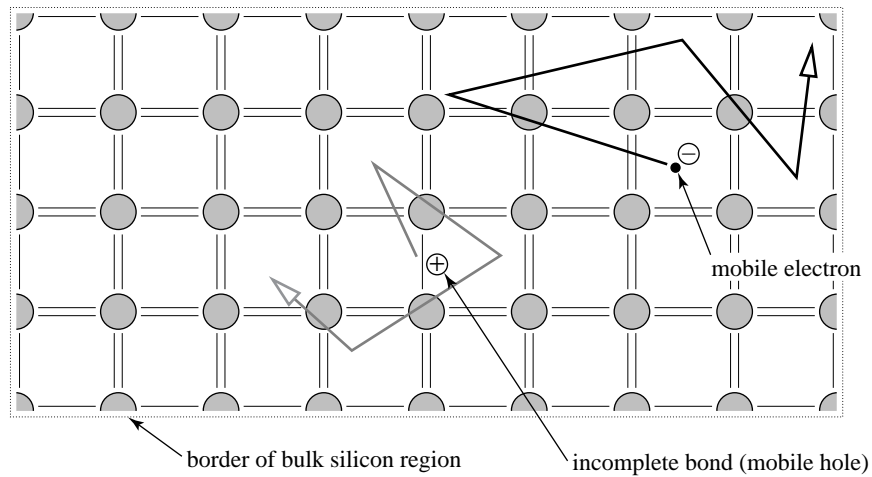


- diamond lattice
- atoms are bonded tetrahedrally with covalent bonds



- Two-dimensional crystal for sketching

## Intrinsic (Pure) Silicon ( $T > 0$ )



- electron: mobile negative unit charge, concentration  $n$  ( $\text{cm}^{-3}$ )
- hole: mobile positive unit charge, concentration  $p$  ( $\text{cm}^{-3}$ )

Unit of charge:  $q = 1.6 \times 10^{-19}$  Coulombs [C]

# Thermal Equilibrium

Generation rate:  $G$  units:  $\text{cm}^{-3} \text{s}^{-1}$  (thermal, optical processes)

Recombination rate:  $R \propto n \cdot p$

$n$  = electron concentration  $\text{cm}^{-3}$

$p$  = hole concentration  $\text{cm}^{-3}$

With the absence of external stimulus,  $G_o = R_o$

*subscript "o" indicates thermal equilibrium*

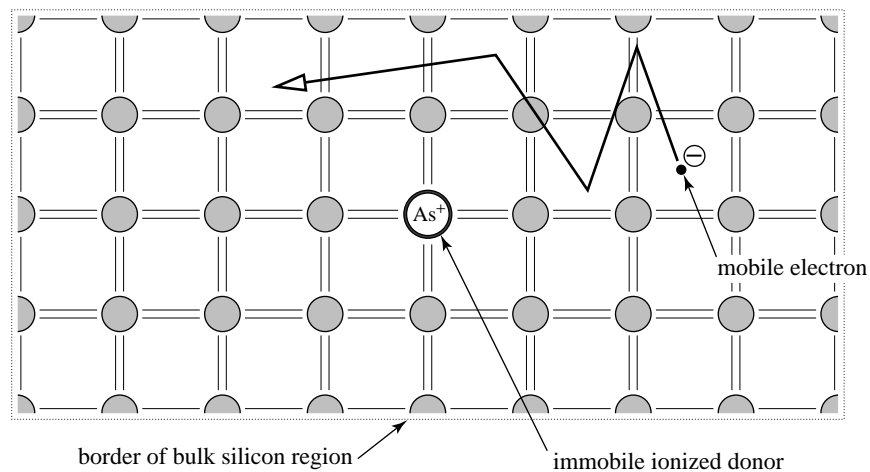
$n_o p_o = \text{constant} = n_i^2 = 10^{20} \text{cm}^{-3}$  at room temperature (approximately)

Since holes and electrons are created *together* in intrinsic silicon,

$n_o = p_o$  which implies that both are equal to  $n_i = 10^{10} \text{cm}^{-3}$

# Doping

Donors (group V) *donate* their 5<sup>th</sup> valence electron and become fixed positive charges in the lattice. Examples: Arsenic, Phosphorus.



How are the thermal equilibrium electron and hole concentrations changed by doping?

- > region is “bulk silicon” -- in the interior of the crystal, away from surfaces
- > charge in region is *zero*, before and after doping:

$$\rho = \text{charge density (C/cm}^3\text{)} = 0 = \underbrace{(-qn_o)}_{\text{electrons}} + \underbrace{(qp_o)}_{\text{holes}} + \underbrace{(qN_d)}_{\text{donors}}$$

where the donor concentration is  $N_d$  ( $\text{cm}^{-3}$ )

## Electron Concentration in Donor-Doped Silicon

Since we are in thermal equilibrium,  $n_o p_o = n_i^2$  (not changed by doping):

Substitute  $p_o = n_i^2 / n_o$  into charge neutrality equation and find that:

$$0 = -qn_o + \frac{qn_i^2}{n_o} + qN_d$$

Quadratic formula -->

$$n_o = \frac{N_d + \sqrt{N_d^2 + 4n_i^2}}{2} = \frac{N_d}{2} + \frac{N_d}{2} \sqrt{1 + \frac{4n_i^2}{N_d^2}}$$

We *always* dope the crystal so that  $N_d \gg n_i$  ... ( $N_d = 10^{13} - 10^{19} \text{ cm}^{-3}$ ), so the square root reduces to 1:

$$n_o = N_d$$

The equilibrium hole concentration is:

$$p_o = n_i^2 / N_d$$

“one electron per donor” is a way to remember the electron concentration in silicon doped with donors.

## Numerical Example

Donor concentration:  $N_d = 10^{15} \text{ cm}^{-3}$

Thermal equilibrium electron concentration:

$$n_o \approx N_d = 10^{15} \text{ cm}^{-3}$$

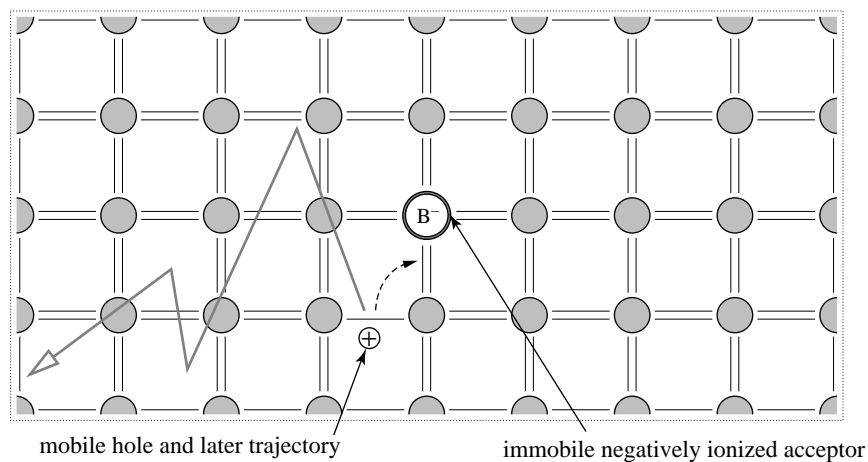
Thermal equilibrium hole concentration:

$$p_o = n_i^2 / n_o \approx n_i^2 / N_d = (10^{10} \text{ cm}^{-3})^2 / 10^{15} \text{ cm}^{-3} = 10^5 \text{ cm}^{-3}$$

Silicon doped with donors is called **n-type** and electrons are the **majority carriers**.  
Holes are the (nearly negligible) **minority carriers**.

## Doping with Acceptors

Acceptors (group III) *accept* an electron from the lattice to fill the incomplete fourth covalent bond and thereby create a mobile hole and become fixed negative charges. Example: Boron.



Acceptor concentration is  $N_a$  ( $\text{cm}^{-3}$ ), we have  $N_a \gg n_i$  typically and so:

one hole is added per acceptor:

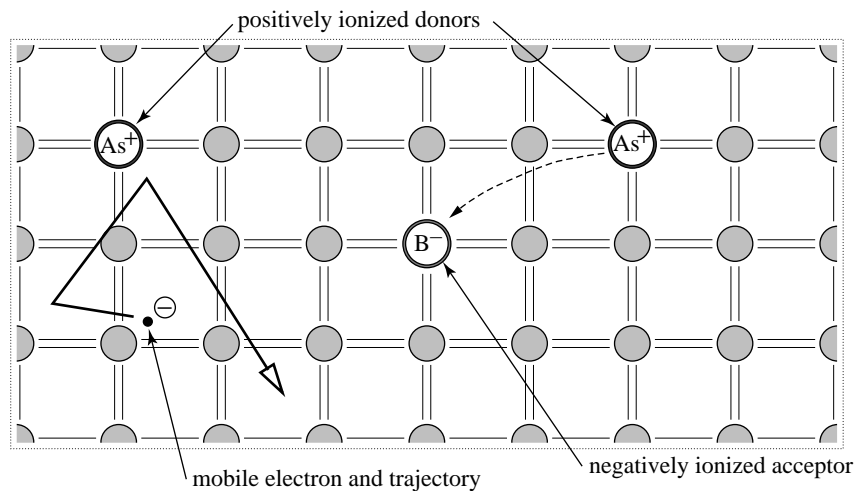
$$p_o = N_a$$

equilibrium electron concentration is:

$$n_o = n_i^2 / N_a$$

## Doping with both Donors and Acceptors: Compensation

- Typical situation is that *both* donors and acceptors are present in the silicon lattice ... mass action law means that  $n_o \neq N_d$  and  $p_o \neq N_a$  !



- Applying charge neutrality with four types of charged species:

$$\rho = -qn_o + qp_o + qN_d - qN_a = q(p_o - n_o + N_d - N_a) = 0$$

we can substitute from the mass-action law  $n_o p_o = n_i^2$  for either the electron concentration or for the hole concentration: which one is the majority carrier?

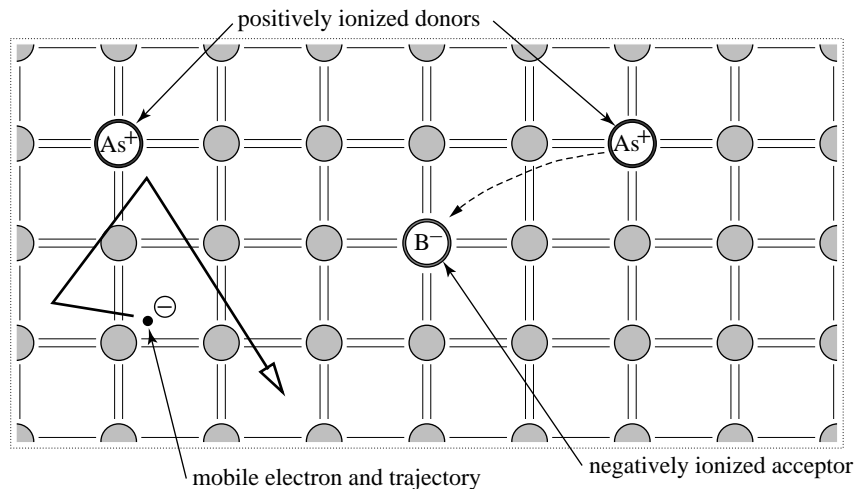
answer (not surprising):  $N_d > N_a \rightarrow$  electrons

$N_a > N_d \rightarrow$  holes



# Compensation

Example shows  $N_d > N_a$



- Applying charge neutrality with four types of charged species:

$$\rho = -qn_o + qp_o + qN_d - qN_a = q(p_o - n_o + N_d - N_a) = 0$$

we can substitute from the mass-action law  $n_o p_o = n_i^2$  for either the electron concentration or for the hole concentration: which one is the majority carrier?

answer (not surprising):  $N_d > N_a \rightarrow$  electrons

$N_a > N_d \rightarrow$  holes

## Carrier Concentrations in Compensated Silicon

- For the case where  $N_d > N_a$ , the electron and hole concentrations are:

$$n_o \cong N_d - N_a \quad \text{and} \quad p_o \cong \frac{n_i^2}{N_d - N_a}$$

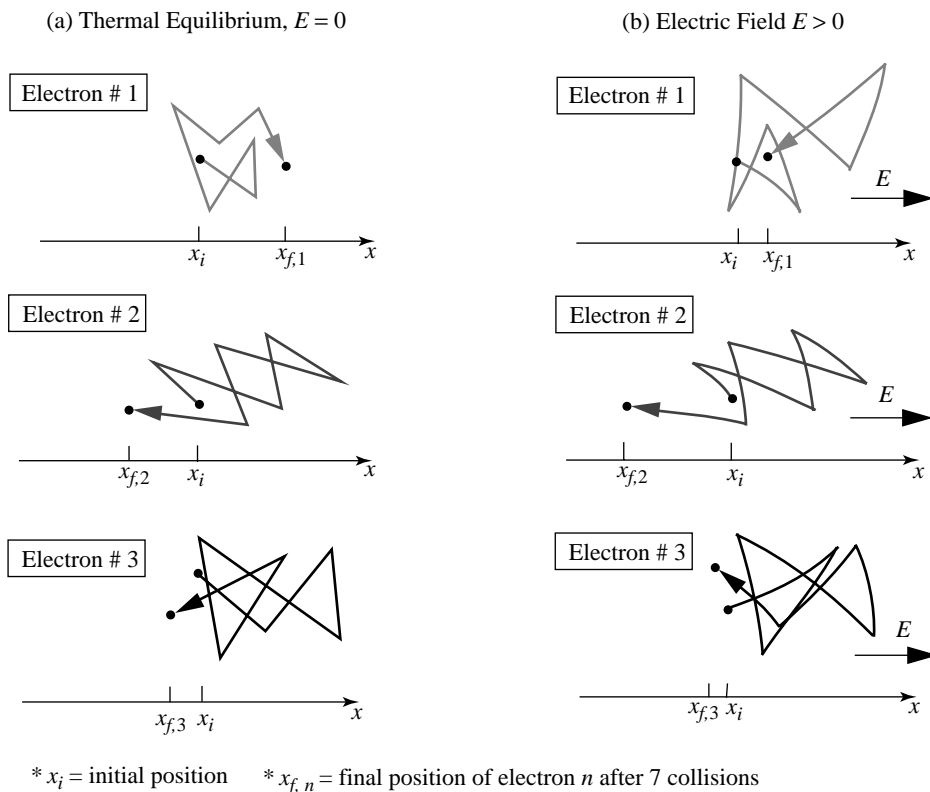
- For the case where  $N_a > N_d$ , the hole and electron concentrations are:

$$p_o \cong N_a - N_d \quad \text{and} \quad n_o \cong \frac{n_i^2}{N_a - N_d}$$

Note that these approximations assume that  $|N_d - N_a| \gg n_i$ , which is nearly always true.

## Carrier Transport: Drift

- If an electric field is applied to silicon, the holes and the electrons “feel” an electrostatic force  $F_e = (+q \text{ or } -q)E$ .
- Picture of effect of electric field on representative electrons: moving at the thermal velocity =  $10^7$  cm/s ... *very fast*, but colliding every  $0.1 \text{ ps} = 10^{-13}$  s. Distance between collisions =  $10^7 \text{ cm/s} \times 10^{-13} \text{ s} = 0.01 \text{ }\mu\text{m}$



- The average of the position changes for the case with  $E > 0$  is  $\overline{\Delta x} < 0$

## Drift Velocity and Mobility

- The *drift velocity*  $v_{dn}$  of electrons is defined as:

$$v_{dn} = \frac{\overline{\Delta x}}{\Delta t}$$

- Experiment shows that the drift velocity is proportional to the electric field for electrons

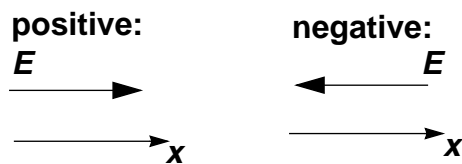
$$v_{dn} = -\mu_n E,$$

with the constant  $\mu_n$  defined as the *electron mobility*.

- Holes drift in the direction of the applied electric field, with the constant  $\mu_p$  defined as the *hole mobility*.

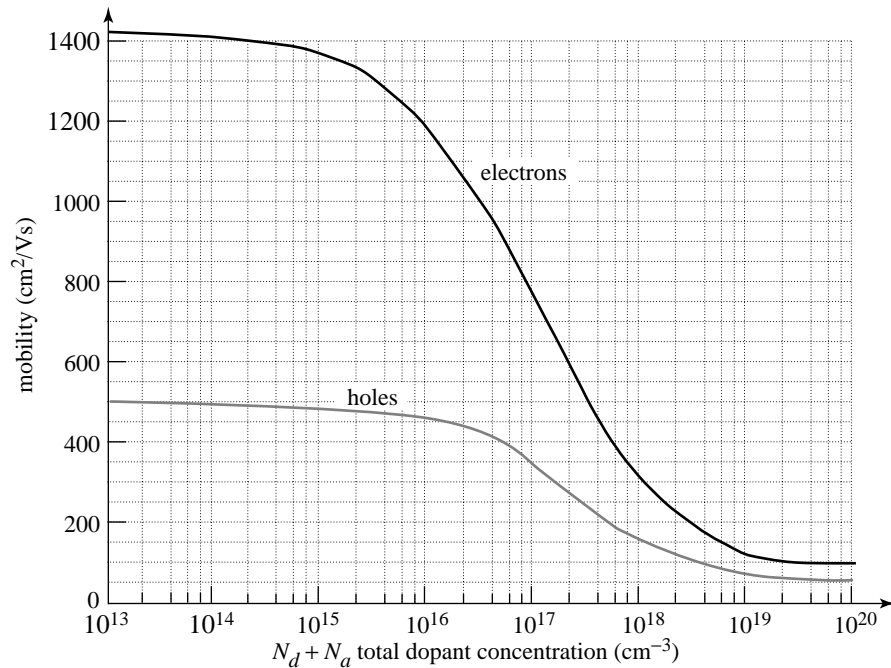
$$v_{dp} = \mu_p E$$

How do we know what's positive and what's negative?



## Electron and Hole Mobilities

- mobilities vary with doping level -- plot is for 300 K = room temp.



- “typical values” for bulk silicon - assuming around  $5 \times 10^{16} \text{ cm}^{-3}$  doping

$$\mu_n = 1000 \text{ cm}^2/(\text{Vs})$$

$$\mu_p = 400 \text{ cm}^2/(\text{Vs})$$

- at electric fields greater than around  $10^4 \text{ V/cm}$ , the drift velocities saturate --> max. out at around  $10^7 \text{ cm/s}$ . Velocity saturation is very common in VLSI devices, due to sub-micron dimensions

## Carrier Transport: Drift Current Density

Electrons drifting opposite to the electric field are carrying negative charge; therefore, the *drift current density* is:

$$J_n^{dr} = (-q) n v_{dn} \quad \text{units: } \text{Ccm}^{-2} \text{ s}^{-1} = \text{Acm}^{-2}$$

$$J_n^{dr} = (-q) n (-\mu_n E) = q n \mu_n E$$

Note that  $J_n^{dr}$  is in the *same* direction as the electric field.

For holes, the mobility is  $\mu_p$  and the drift velocity is in the same direction as the electric field:  $v_{dp} = \mu_p E$

The hole drift current density is:

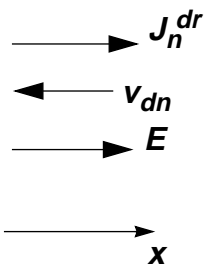
$$J_p^{dr} = (+q) p v_{dp}$$

$$J_p^{dr} = q p \mu_p E$$

## Drift Current Directions and Signs

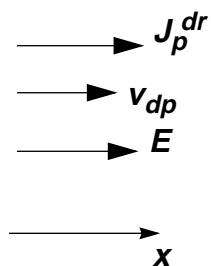
- For electrons, an electric field in the  $+x$  direction will lead to a drift velocity in the  $-x$  direction ( $v_{dn} < 0$ ) and a drift current density in the  $+x$  direction ( $J_n^{dr} > 0$ ).

**electron drift  
current density**



- For holes, an electric field in the  $+x$  direction will lead to a drift velocity in the  $+x$  direction ( $v_{dp} > 0$ ) and a drift current density in the  $+x$  direction ( $J_p^{dr} > 0$ ).

**hole drift  
current density**



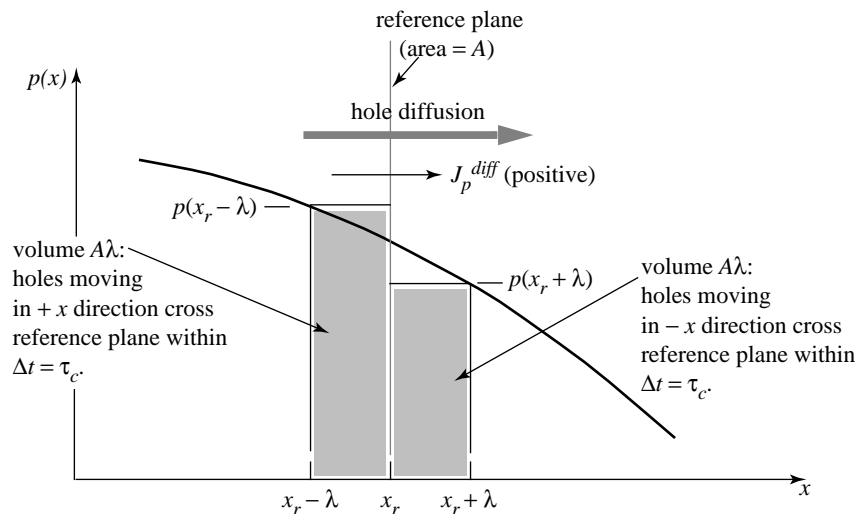
# Carrier Transport: Diffusion

*Diffusion* is a transport process driven by gradients in the concentration of particles in random motion and undergoing frequent collisions -- such as ink molecules in water ... or holes and electrons in silicon.

Mathematics: find the number of carriers in a volume  $A\lambda$  on either side of the reference plane, where  $\lambda$  is the mean free path between collisions.

- Some numbers: average carrier velocity =  $v_{th} = 10^7$  cm/s, average interval between collisions =  $\tau_c = 10^{-13}$  s = 0.1 picoseconds

mean free path =  $\lambda = v_{th} \tau_c = 10^{-6}$  cm = 0.01  $\mu$ m



- half of the carriers in each volume will pass through the plane before their next collision, since their motion is random



## Carrier Transport: Diffusion Current Density

- Current density = (charge) x (# carriers per second per area):

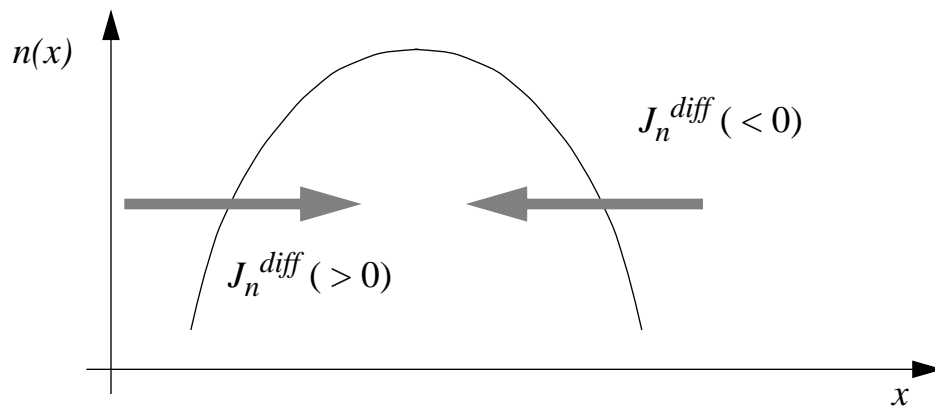
$$J_p^{diff} = q \left[ \frac{\frac{1}{2}p(x-\lambda)A\lambda - \frac{1}{2}p(x+\lambda)A\lambda}{A\tau_c} \right]$$

- If we assume that  $\lambda$  is much smaller than the dimensions of our device, then we can consider  $\lambda = dx$  and use Taylor expansions :

$$J_p^{diff} = -qD_p \frac{dp}{dx}, \quad \text{where } D_p = \lambda^2 / \tau_c \text{ is the diffusion coefficient}$$

## Electron Transport by Diffusion

- Electrons diffuse down the concentration gradient, yet carry negative charge --> electron diffusion current density points in the direction of the gradient



- Total current density: add drift and diffusion components for electrons and for holes --

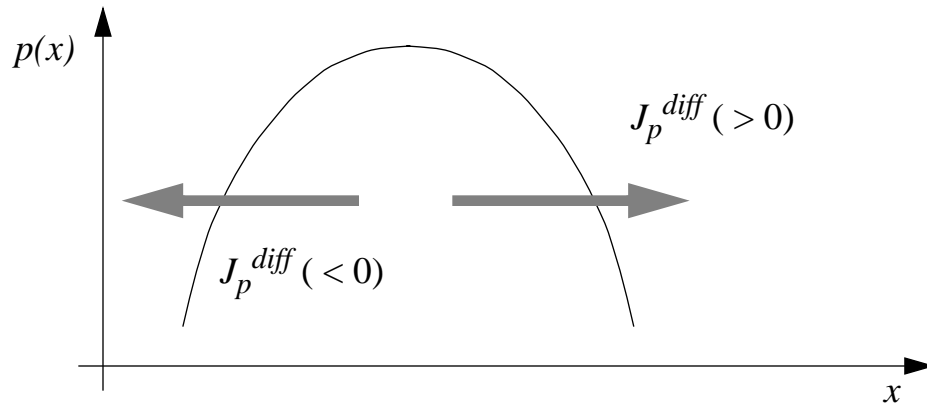
$$J_n = J_n^{dr} + J_n^{diff} = qn\mu_n E + qD_n \frac{dn}{dx}$$

$$J_p = J_p^{dr} + J_p^{diff} = qp\mu_p E - qD_p \frac{dp}{dx}$$

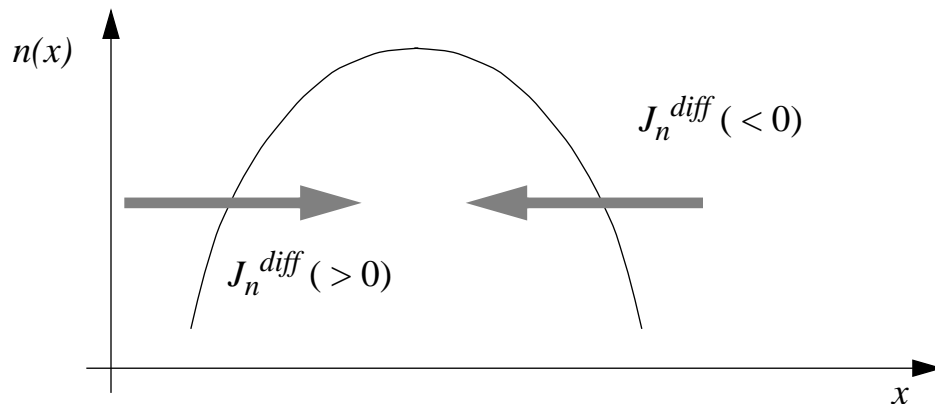
- Fortunately, we will be able to eliminate one or the other component in finding the internal currents in microelectronic devices.

## Carrier Transport by Diffusion

- Holes diffuse “down” the concentration gradient and carry a positive charge --> hole diffusion current has the *opposite* sign to the gradient in hole concentration  $dp/dx$



- Electrons diffuse down the concentration gradient, yet carry a negative charge --> electron diffusion current density has the *same* sign as the gradient in electron concentration  $dn/dx$ .



## Electron Diffusion Current Density

- Similar analysis leads to

$$J_n^{diff} = qD_n \frac{dn}{dx},$$

where  $D_n$  is the electron diffusion coefficient (units:  $\text{cm}^2/\text{s}$ )

- Numerical values of diffusion coefficients: use Einstein's relation

$$\frac{D_n}{\mu_n} = \frac{kT}{q}$$

- The quantity  $kT/q$  has units of volts and is called the *thermal voltage*,  $V_{th}$ :

$$V_{th} = \frac{kT}{q} = 25 - 26 \text{ mV},$$

at “room temperature,” with 25 mV for a cool room (62 °F) and 26 mV for a warm room (83 °F).

We will pick 25 mV or 26 mV depending on which gives the “roundest” numbers.

## Total Current Densities

- Add drift and diffusion components for electrons and for holes --

$$J_n = J_n^{dr} + J_n^{diff} = qn\mu_n E + qD_n \frac{dn}{dx}$$

$$J_p = J_p^{dr} + J_p^{diff} = qp\mu_p E - qD_p \frac{dp}{dx}$$

- Fortunately, we will be able to eliminate one or the other component of the electron or the hole current in our analysis of semiconductor devices.