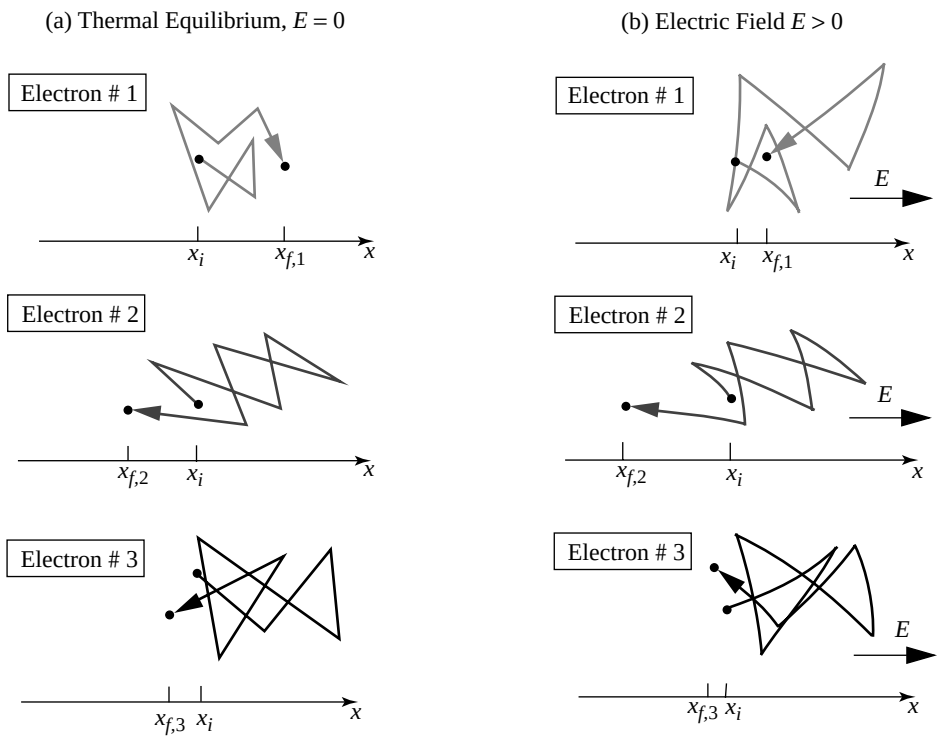
Carrier Transport: Drift

- If an electric field is applied to silicon, the holes and the electrons "feel" an electrostatic force $F_e = (+q \text{ or } -q)E$.
- Picture of effect of electric field on representative electrons: moving at the thermal velocity = 10^7 cm/s ... *very fast*, but colliding every 0.1 ps = 10^{-13} s. Distance between collsions = 10^7 cm/s x 10^{-13} cm = 0.01 μm



* x_i = initial position * $x_{f,n}$ = final position of electron n after 7 collisions

- The average of the position changes for the case with $E > 0$ is $\overline{\Delta x} < 0$

Drift Velocity and Mobility

- The *drift velocity* v_{dn} of electrons is defined as:

$$v_{dn} = \frac{\overline{\Delta x}}{\Delta t}$$

- Experiment shows that the drift velocity is proportional to the electric field for electrons

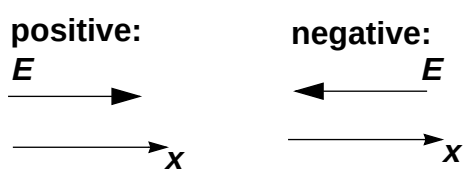
$$v_{dn} = -\mu_n E,$$

with the constant μ_n defined as the *electron mobility.*

- Holes drift in the direction of the applied electric field, with the constant μ_p defined as the *hole mobility.*

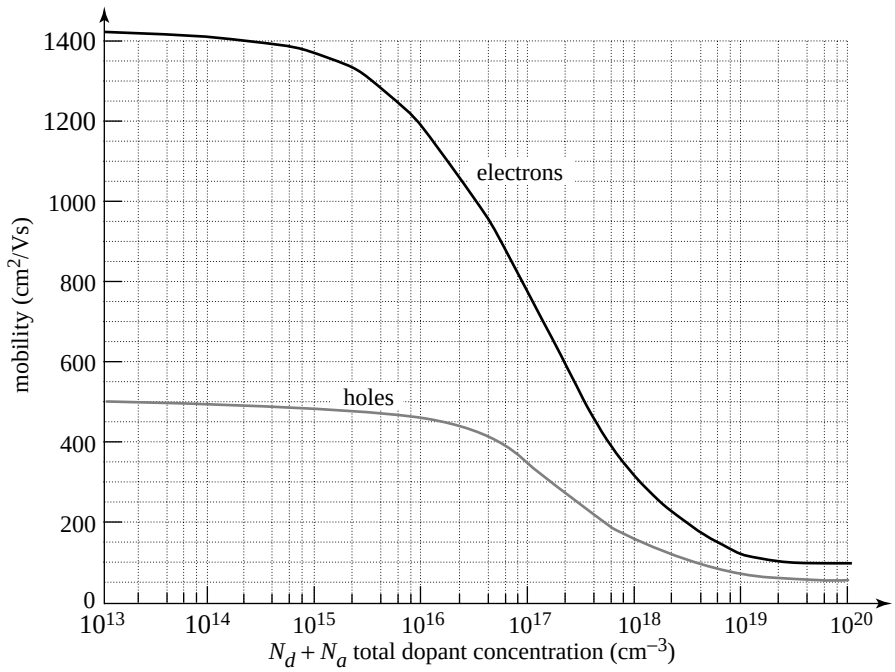
$$v_{dp} = \mu_p E$$

How do we know what's positive and what's negative?



Electron and Hole Mobilities

- mobilities vary with doping level -- plot is for 300 K = room temp.



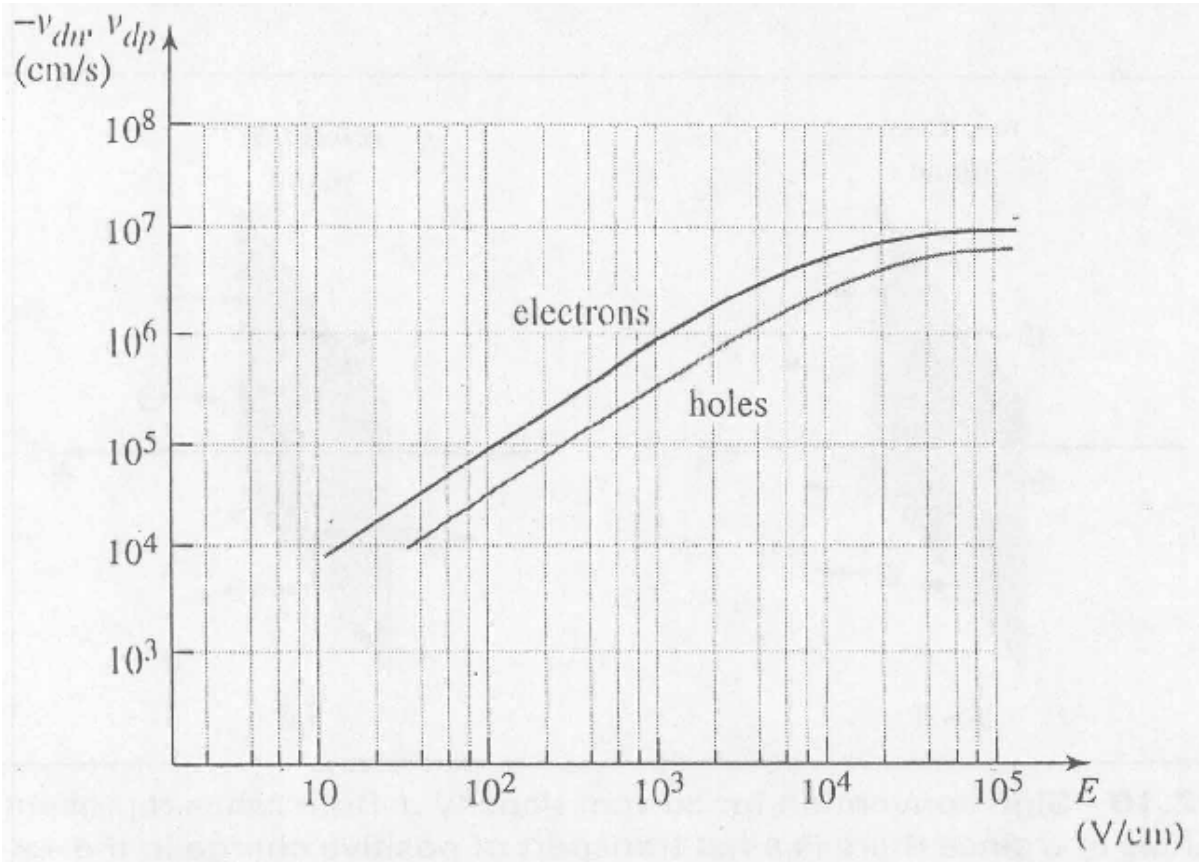
- "typical values" for bulk silicon - assuming around 5 x 10^{16} cm^{-3} doping

$$\mu_n = 1000\ cm^2/(Vs)$$

$$\mu_p = 400\ cm^2/(Vs)$$

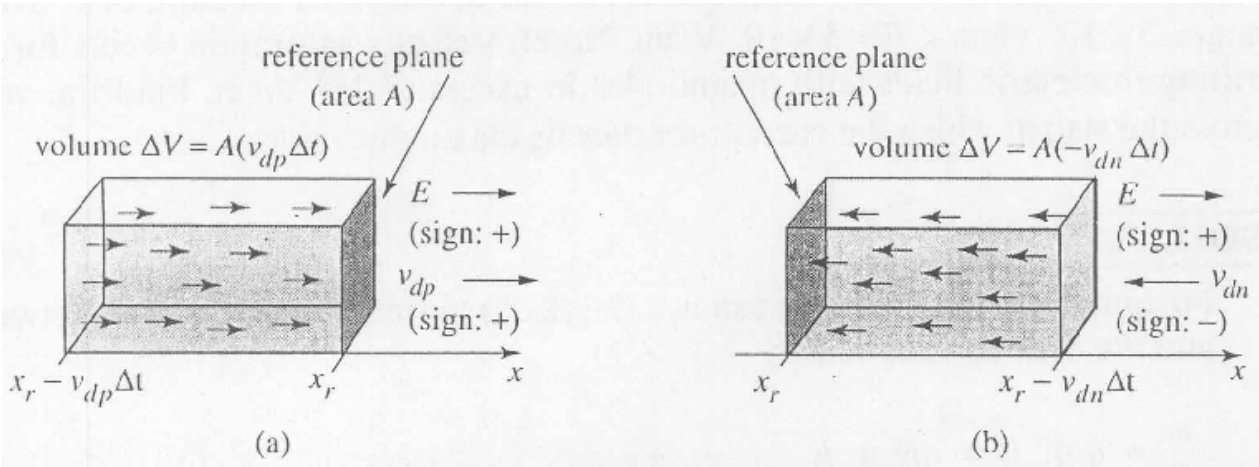
- at electric fields greater than around 10^4 V/cm, the drift velocities saturate --> max. out at around 10^7 cm/s. Velocity saturation is very common in VLSI devices, due to sub-micron dimensions

Drift velocity saturation



- at electric fields greater than around 10^4 V/cm, the drift velocities saturate --> max. out at around 10^7 cm/s. Velocity saturation is very common in VLSI devices, due to sub-micron dimensions

Drift current density



In a time Δt, the volume is $\Delta V = A(v_{dp}\Delta t)$

Number of holes *Np* crossing the plane in Δt

$$\Delta N_p = p\Delta V$$

The hole charge drifting across the reference plane ΔQ_p is the product of ΔN_p and the charge per hole q. According to the definition of current density

$$J_p^{dr} = \frac{\Delta Q_p}{A\Delta t} = \frac{qp\Delta V}{A\Delta t} = \frac{qpAv_{dp}\Delta t}{A\Delta t} = qpv_{dp}$$

Carrier Transport: Drift Current Density

Electrons drifting opposite to the electric field are carrying negative charge; therefore, the *drift current density* is:

$$J_n^{dr} = (-q)\, n\, v_{dn} \qquad \text{units: Ccm}^{-2}\ \text{s}^{-1} = \text{Acm}^{-2}$$

$$\boxed{J_n^{dr} = (-q)\, n\, (-\mu_n E) = q\, n\, \mu_n E}$$

Note that J_n^{dr} is in the *same* direction as the electric field.

For holes, the mobility is μ_p and the drift velocity is in the same direction as the electric field: $v_{dp} = \mu_p E$

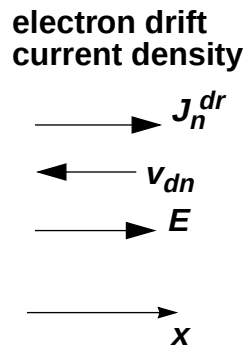
The hole drift current density is:

$$J_p^{dr} = (+q)\, p\, v_{dp}$$

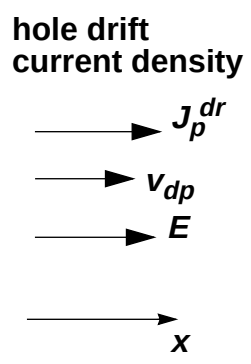
$$\boxed{J_p^{dr} = q\, p\, \mu_p E}$$

Drift Current Directions and Signs

- For electrons, an electric field in the $+x$ direction will lead to a drift velocity in the $-x$ direction ($v_{dn} < 0$) and a drift current density in the $+x$ direction ($J_n^{dr} > 0$).



- For holes, an electric field in the $+x$ direction will lead to a drift velocity in the $+x$ direction ($v_{dp} > 0$) and a drift current density in the $+x$ direction ($J_n^{dr} > 0$).



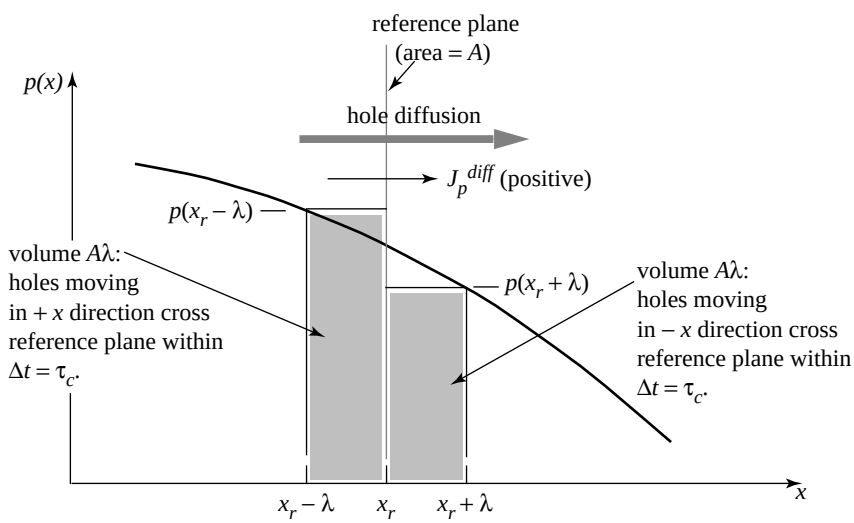
Carrier Transport: Diffusion

Diffusion is a transport process driven by gradients in the concentration of particles in random motion and undergoing frequent collisions -- such as ink molecules in water ... or holes and electrons in silicon.

Mathematics: find the number of carriers in a volume $A\lambda$ on either side of the reference plane, where λ is the mean free path between collisions.

- Some numbers: average carrier velocity = $v_{th} = 10^7$ cm/s, average interval between collisions = $\tau_c = 10^{-13}$ s = 0.1 picoseconds

  mean free path = $\lambda = v_{th}\,\tau_c = 10^{-6}$ cm = 0.01 μm



- half of the carriers in each volume will pass through the plane before their next collision, since their motion is random

Carrier transport:
Diffusion current density

- Current density = (charge) x (# carriers per second per area):

$$J_p^{diff} = q\left[\frac{\frac{1}{2}p(x-\lambda)A\lambda - \frac{1}{2}p(x+\lambda)A\lambda}{A\tau_c}\right]$$

- If we assume that λ is much smaller than the dimensions of our device, then we can consider $\lambda = dx$ and use Taylor expansions :

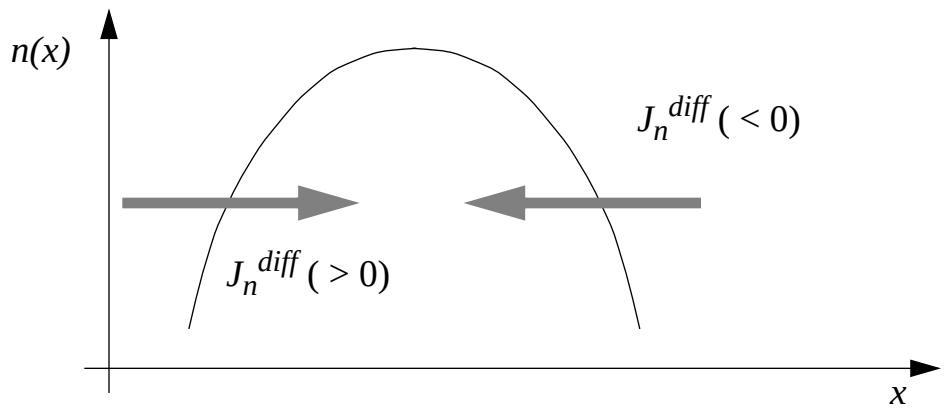
$$J_p^{diff}(x = x_r) = \frac{q\lambda}{\tau_c}\left(\frac{\left(p(x_r) - \left.\frac{dp}{dx}\right|_{x_r}\lambda\right) - \left(p(x_r) + \left.\frac{dp}{dx}\right|_{x_r}\lambda\right)}{2}\right)$$

$$J_p^{diff} = -qD_p\frac{dp}{dx}\ ,$$ where $D_p = \lambda^2 / \tau_c$ is the diffusion coefficent

$$J_n^{diff} = qD_n\frac{dn}{dx},$$ for electrons

Electron Transport by Diffusion

- Electrons diffuse down the concentration gradient, yet carry negative charge --> electron diffusion current density points in the direction of the gradient



- Total current density: add drift and diffusion components for electrons and for holes --

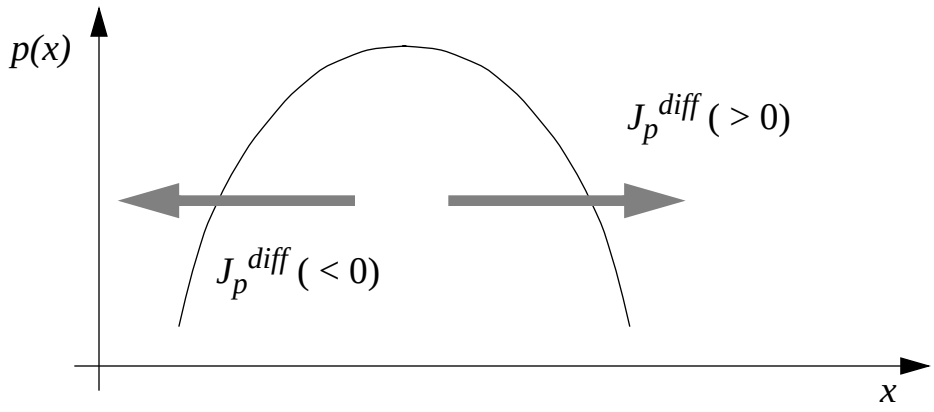
$$J_n = J_n^{dr} + J_n^{diff} = qn\mu_n E + qD_n \frac{dn}{dx}$$

$$J_p = J_p^{dr} + J_p^{diff} = qp\mu_p E - qD_p \frac{dp}{dx}$$

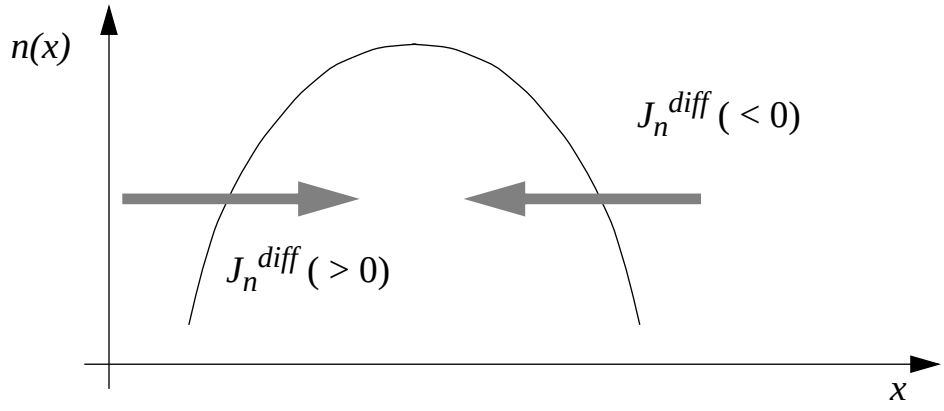
- Fortunately, we will be able to eliminate one or the other component in finding the internal currents in microelectronic devices.

Carrier Transport by Diffusion

- Holes diffuse “down” the concentration gradient and carry a positive charge --> hole diffusion current has the *opposite* sign to the gradient in hole concentration *dp/dx*



- Electrons diffuse down the concentration gradient, yet carry a negative charge --> electron diffusion current density has the *same* sign as the gradient in electron concentration *dn/ dx*.



Electron Diffusion Current Density

- Similar analysis leads to

$$J_n^{diff} = qD_n \frac{dn}{dx},$$

where D_n is the electron diffusion coefficient (units: cm^2/s)

- Numerical values of diffusion coefficients: use Einstein's relation

$$\boxed{\frac{D_n}{\mu_n} = \frac{kT}{q}}$$

- The quantity kT/q has units of volts and is called the *thermal voltage*, V_{th}:

$$V_{th} = \frac{kT}{q} = 25 - 26\,\text{mV},$$

at "room temperature," with 25 mV for a cool room (62 ^{o}F) and 26 mV for a warm room (83 ^{o}F).

We will pick 25 mV or 26 mV depending on which gives the "roundest" numbers.

Total Current Densities

- Add drift and diffusion components for electrons and for holes --

$$J_n = J_n^{dr} + J_n^{diff} = qn\mu_n E + qD_n \frac{dn}{dx}$$

$$J_p = J_p^{dr} + J_p^{diff} = qp\mu_p E - qD_p \frac{dp}{dx}$$

- Fortunately, we will be able to eliminate one or the other component of the electron or the hole current in our analysis of semiconductor devices.