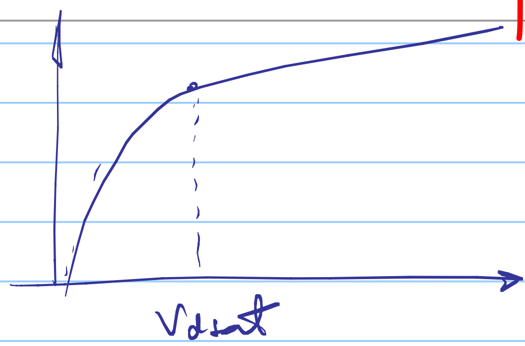
Note Title 12/5/2006

CALCULO DE Req.



Modelo: 1) $I_D = I_{Dsat}$.

2) $I_D = I_{Dsat} \cdot (1 + \lambda V_{DS})$.

Cálculo de Req de acuerdo a tp.

a) La descarga del cap. C_L es

$$I_{Dsat} = C_L \cdot \frac{V_{dd}}{2 \cdot t_p} \qquad t_p = \frac{V_{dd}}{2 I_{Dsat}} \cdot C_L$$

La descarga de un circuito RC es

$$V(t) = V(t_0) \cdot e^{-t/RC}$$

Si $V(t) = V(t_0)/2 \longrightarrow \frac{1}{2} = e^{-t_p/RC}$

$$e^{t_p/RC} = 2 \qquad \frac{t_p}{RC} = \ln(2) \qquad t_p = \ln(2) RC.$$

Luego $\frac{V_{dd}}{2 I_{Dsat}} = \ln(2) R_{eq}$

$$R_{eq} = \frac{1}{2 \ln(2)} \cdot \frac{V_{dd}}{I_{Dsat}}.$$

Supongamos ahora $I_D = I_{DSAT} \cdot (1 + \lambda \cdot (V_{DS} - V_{DSAT}))$

$$I_D = C_L \cdot \frac{dV_O}{dt} \rightarrow \quad dt = \frac{C_L}{I_{DSAT}} \cdot \frac{dV_O}{(1 + \lambda \cdot (V_{DS} - V_{DSAT}))}.$$

$$\int_{t_i}^{t_f} dt = t_p = \frac{C_L}{I_{DSAT}} \int_{V_{DD}/2}^{V_{DD}} \frac{1}{1 + \lambda \cdot (V_O - V_{DSAT})} \cdot dV_O.$$

$$t_p = \frac{C_L}{I_{DSAT}} \cdot \left. \frac{\log (1 + \lambda \cdot (V_O - V_{DSAT}))}{\lambda} \right|_{V_{DD}/2}^{V_{DD}}$$

$$t_p = \frac{C_L}{I_{DSAT} \lambda} \cdot \log \frac{1 + \lambda \cdot (V_{DD} - V_{DSAT})}{1 + \lambda \left(\frac{V_{DD}}{2} - V_{DSAT} \right)}$$

$$= \frac{C_L}{\lambda I_{DSAT}} \cdot \log \frac{I_{DSAT} (1 + \lambda (V_{DD} - V_{DSAT}))}{I_{DSAT} (1 + \lambda (\frac{V_{DD}}{2} - V_{DSAT}))}$$

$$= \frac{C_L}{\lambda I_{DSAT}} \cdot \log \frac{I_D(V_{DD})}{I_D(V_{DD}/2)}.$$

Cálculo de λ.

$$\lambda = \frac{I_D(V_{DD}) - I_D(V_{DD}/2)}{I_{DSAT}} \cdot \frac{1}{V_{DD}/2}.$$

$$t_p = \frac{C_L \cdot (V_{DD}/2)}{I_D(V_{DD}) - I_D(V_{DD}/2)} \cdot \log \frac{I_D(V_{DD})}{I_D(V_{DD}/2)}$$

$$t_p = \frac{1}{2} \frac{C_L \cdot V_{DD}}{(I_D^2 - I_D^1)} \cdot \log \frac{I_D^2}{I_D^1}$$

$$R_{eq} = \frac{1}{2 \times \ln(2)} \cdot \frac{V_{DD}}{I_D^2 - I_D^1} \cdot \log \frac{I_D^2}{I_D^1}$$

En forma analítica

$$t_p = \frac{C_L}{\lambda \cdot K \cdot S \cdot \left(V_{DD} - V_T - \frac{V_{DSAT}}{2}\right) V_{DSAT}} \log \frac{I_D(V_{DD})}{I_{DD}(V_{DD}/2)}$$

o

$$t_p = \frac{C_L}{\lambda \cdot K \cdot S \left(V_{DD} - V_T - \frac{V_{DSAT}}{2}\right) V_{DSAT}} \times \log \frac{1 + \lambda(V_{DD} - V_{DSAT})}{1 + \lambda\left(\frac{V_{DD}}{2} - V_{DSAT}\right)}$$

Ejemplo: AMI 0,5 μm. $\frac{W}{L} = \frac{1.8\,\mu m}{0.6\,\mu m}$. $C_L = 100\,fF$

SPICE: $I_D(V_{DD}) = 568\,\mu A$

$I_D\left(\frac{V_{DD}}{2}\right) = 535\,\mu A$

$t_{p(H-L)} = 483\,ps.$

Cálculo $t_p = 453\,ps.$

Nota: 23 ps son de error debido al sobrepico de la simulación. $t_{pHL}(\text{correg}) = 460\,ps.$

Si tomamos $I'_{D_{SAT}} = \frac{I_{DSAT}}{2} \cdot \left((1+\lambda V_{DD}) + \left(1+\frac{\lambda V_{DD}}{2}\right) \right)$

$$= I_{Dsat} \cdot \left(\frac{1}{2} + \frac{1}{2}\lambda V_{DD} + \frac{1}{2} + \frac{1}{4}\lambda V_{DD} \right) =$$

$$= I_{Dsat} \left(1 + \lambda \cdot V_{DD} \left(\frac{1}{2} + \frac{1}{4} \right) \right) =$$

$$= I_{Dsat} \cdot \left(1 + \frac{3}{4} \cdot \lambda \cdot V_{DD} \right).$$

y

$$R_{eq} = \frac{1}{2\ln(2)} \cdot \frac{V_{DD}}{I_{DSAT} \cdot \left(1 + \frac{3}{4}\lambda V_{DD}\right)}$$

Tomemos la expresión de I_{Dsat}

$$I_{Dsat} = \mu C_{ox} \frac{W}{L} \cdot V_{DSAT} \left(V_{GS} - V_T - \frac{V_{DSAT}}{2} \right)$$

y evaluamos para $V_{GS} = V_{DD}$.

$$I'_{D_{SAT}} = 2 \cdot k'_N \cdot V_{DSAT} \left(V_{DD} - V_T - \frac{V_{DSAT}}{2} \right)$$

y

$$R_{eq} = \frac{1}{2\ln(2)} \cdot \frac{V_{DD}}{2k'_N V_{DSAT} \left(V_{DD} - V_T - \frac{V_{DSAT}}{2} \right) \cdot \left(1 + \frac{3}{4}\lambda V_{DD} \right)}$$

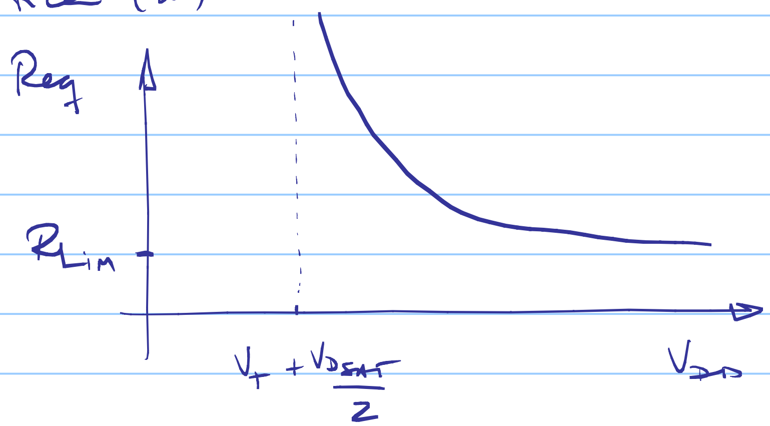
Si es que se tiene en cuenta λ.

Caso contrario

$$R_{eq} = \frac{1}{2\ln(2)} \cdot \frac{V_{DD}}{k'_N V_{DSAT}\left(V_{DD} - V_T - \frac{V_{DSAT}}{2}\right)} \quad (a)$$

Gráfica (a)

Válido solo para $V_G > V_T + \frac{V_{DSAT}}{2}$.



$$R_{Lim} = \frac{1}{2\ln(2)} \cdot \frac{1}{k'_N V_{DSAT}}$$

Si utilizamos la expresión exacta

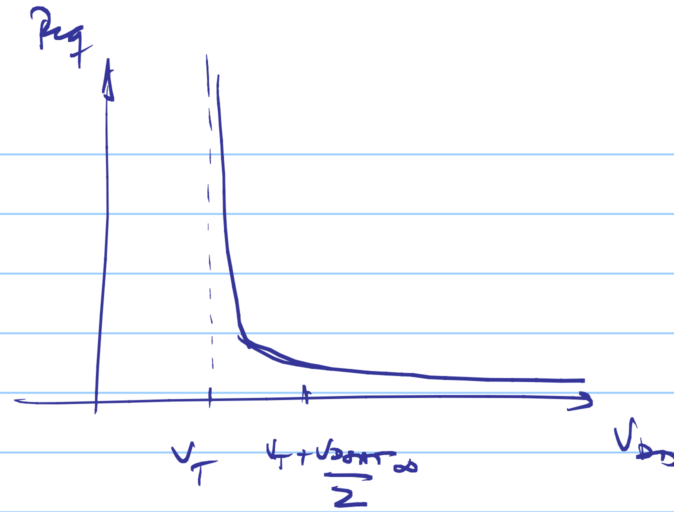
$$R_{eq} = \frac{1}{2\ln(2)} \cdot \frac{V_{DD}}{2K.S\left(V'_G - \frac{V_{DS}}{2}\right) V_{DS}} \cdot \frac{1}{\left(1 + \frac{V_{DS}}{E_c.L}\right)}$$

con $V_{DS} = \frac{E_c L . V'_G}{E_c L + V'_G}$

$$R_{eq} = \frac{1}{2\ln(2)} \cdot \frac{V_{DD}}{2K.S.\left(V_{DD} - V_T - \frac{E_c L.(V_{DD}-V_T)}{2(E_c L + V_{DD} - V_T)}\right)} \times$$

$$\times \frac{1}{\left(\frac{E_c.L\,(V_{DD}-V_T)}{E_c L + V_{DD} - V_T}\right)} \cdot \frac{1}{\left(1 + \frac{V_{DD}-V_T}{E_c L + V_{DD} - V_T}\right)}$$

La gráfica exacta:



En $V_{DD} = V_T + \frac{E_c L}{2}$; $V_G' = V_{DD} - V_T = E_c L/2$.

$$V_{DS} = \frac{E_c L \cdot V_G'}{E_c L + V_G'} = \frac{E_c L \cdot E_c L}{\left(2\frac{E_c L}{2} + \frac{E_c L}{2}\right)} \cdot \frac{1}{2} = \frac{\not{2}}{3}\frac{1}{\not{2}} \cdot E_c L$$

$$\boxed{V_{DSAT} = \frac{E_c L}{3}}$$

$$R_{eq} = \frac{1}{2\ln(2)} \cdot \frac{V_{DD}}{2kS\left(V_{DD} - \frac{E_c L}{2\cdot 3}\right)\frac{E_c L}{3}\cdot\left(1 + \frac{E_c L}{3 \cdot E_c L}\right)}$$

$$= \frac{1}{2\ln(2)} \cdot \frac{V_{DD}}{2kS\left(V_{DD} - \frac{E_c L}{6}\right)\cdot\frac{E_c L}{3}\cdot\frac{4}{3}}.$$

$V_{DD} = V_T + \frac{E_c L}{2}$; $$R_{eq} = \frac{1}{2\ln(2)} \cdot \frac{V_T + E_c L/2}{2kS\left(V_T - \frac{1}{3}E_c L\right)\cdot\frac{4}{9}E_c L} \neq \infty$$