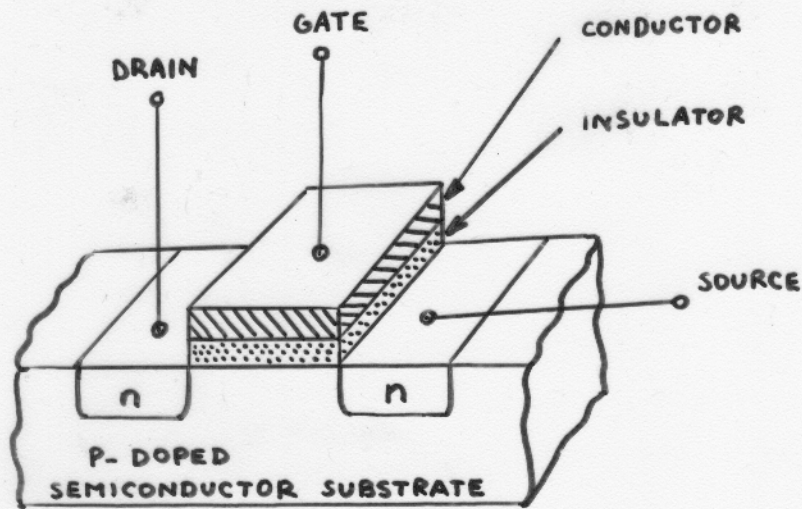
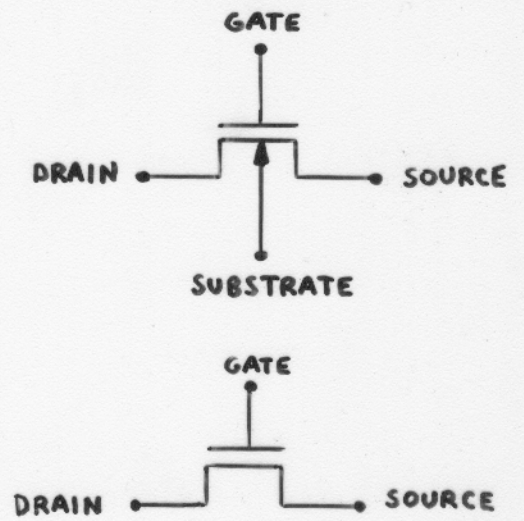


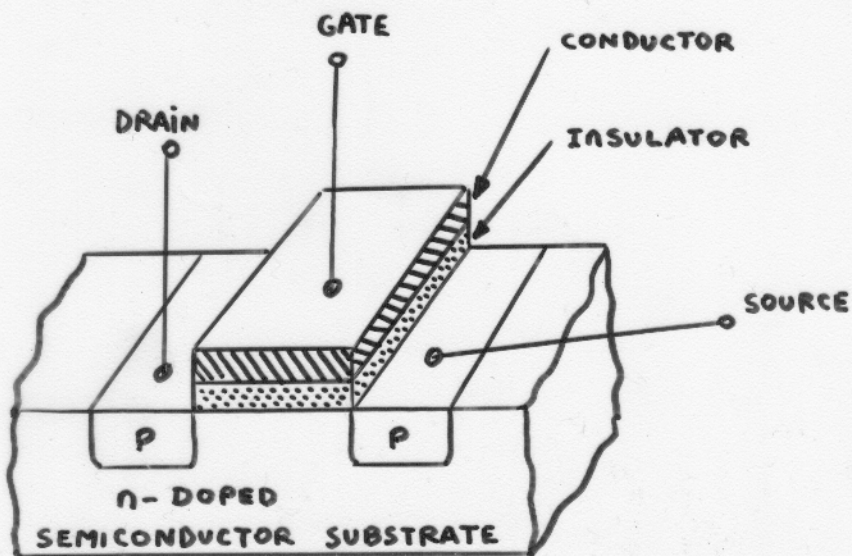
MOS transistor



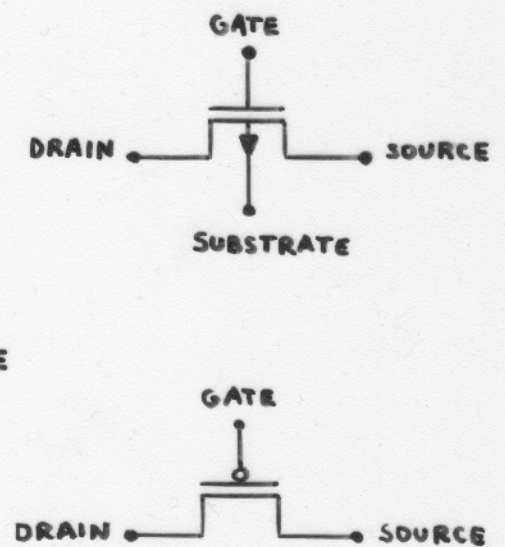
n-Transistor



Circuit symbols



p-Transistor



Circuit symbols

Model to analyze MOS Transistor circuits

The drain current flowing can be calculated as follows:

$$I_D = \frac{\text{charge induced in channel}}{\text{transit time}} = \frac{Q}{\tau}$$

$$Q = C_g V_{cex}$$

C_g = Gate capacity

V_{cex} = Gate voltage in excess of the Threshold voltage.

For a parallel plate capacitor

$$C_g = C_{ox} A$$

C_{ox} = Capacity per unit area of the gate

A = Gate or channel surface area

$$C_{ox} = \frac{\epsilon}{t_{ox}} \quad \begin{array}{l} \text{Permittivity of the oxide dielectric} \\ \text{the oxide thickness} \end{array}$$

$$A = W (\text{width}) \cdot L (\text{length})$$

$$V_{cex} = V_g - V_T$$

$$\tau = \frac{\text{distance electrons have to move}}{\text{average velocity}} = \frac{L}{v}$$

$$v = \mu_n E$$

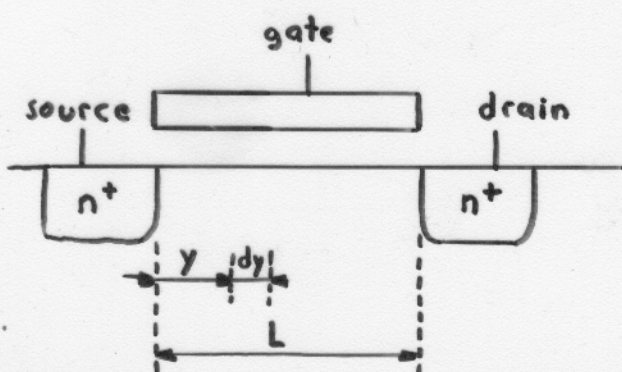
μ_n = mobility of electrons

E = electric field

$$E = \frac{V_d - V_s}{L} \quad ; \quad V_{Ds} < 100 \text{ mV}$$

$$I_D = \frac{C_g (V_g - V_T)}{L / \mu_n E} = \mu_n \frac{C_g}{L^2} (V_g - V_T) \cdot (V_d - V_s)$$

$$I_D = \mu_n C_{ox} \frac{W}{L} (V_g - V_T) \cdot (V_D - V_S)$$



$$Q = C_{ox} W dy (V_g - V_T)$$

$$\tau = \frac{dy}{\mu_n dV/dy}$$

Model to analyze MOS Transistor circuits

$$I_D dy = \mu_n C_{ox} W (V_g - V_T) dv$$

$$I_D = \mu_n C_{ox} \frac{W}{L} \int_{V_s}^{V_D} (V_g - V_T) dV \quad (1)$$

$$\rightarrow (V_D = V_S = 0)$$

$$V_T = V_{FB} - \frac{1}{C_{ox}} \sqrt{2q\epsilon_{si}N'} \sqrt{2|\phi_B|} - 2\phi_B$$

V_{FB} = Flat band voltage q = charge on an electron

ϵ_{si} = dielectric constant of the silicon substrate

N = concentration density of the substrate

$$\phi_B = V_T \ln \frac{N_D}{n_i} \rightarrow 10^{15} \dots 10^{16}$$

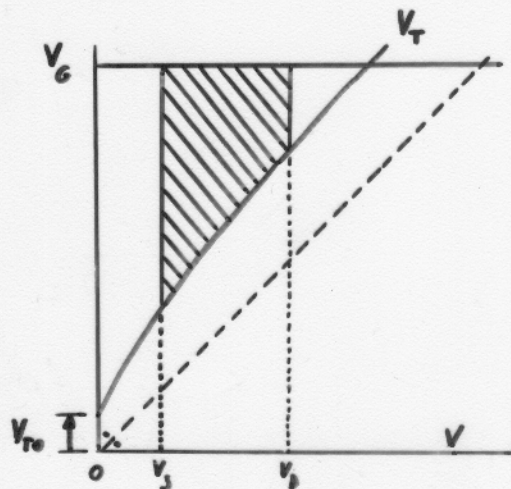
$n_i \rightarrow 10^{10}$

$$\rightarrow (V_D \neq V_S)$$

$$V_T = V_{FB} - \frac{1}{C_{ox}} \sqrt{2q\epsilon_{si}N'} \sqrt{2|\phi_B| + V} - 2\phi_B + V \quad (2)$$

integrating

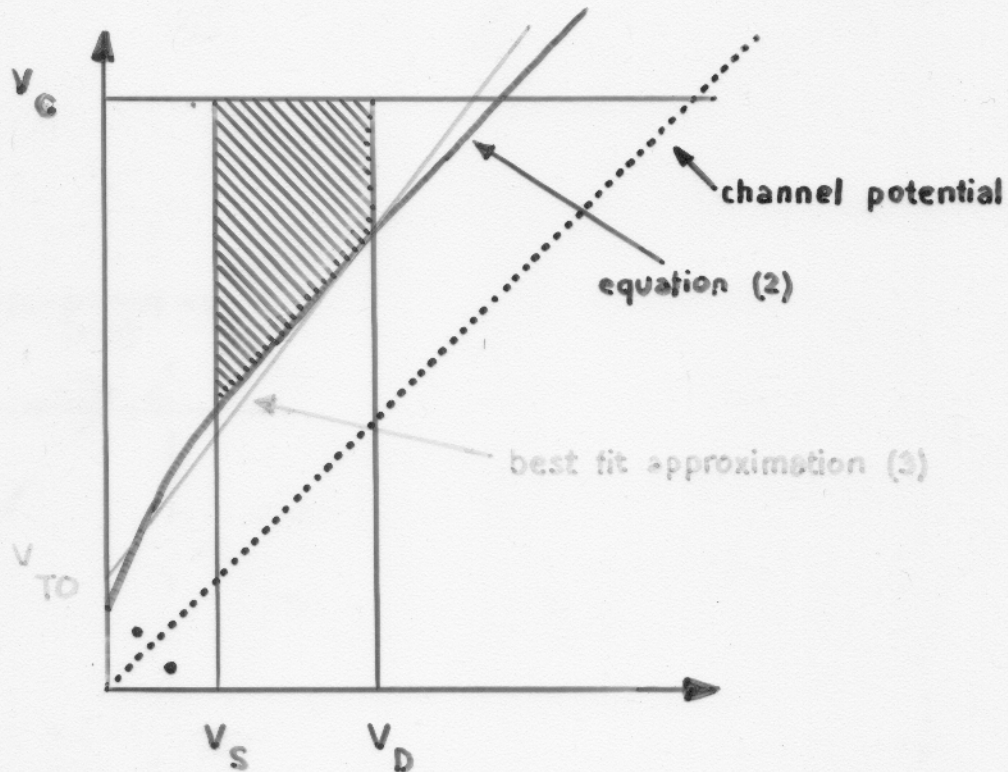
$$I_D = \mu_n C_{ox} \frac{W}{L} \left\{ \left[(V_G - V_{T0}) V_D - \frac{V_D^2}{2} \right] - \frac{2}{3} \frac{\sqrt{2q\epsilon_{si}N'}}{C_{ox}} \left[(V_D + 2|\phi_B|)^{3/2} - (2|\phi_B|)^{3/2} \right] \right\}$$



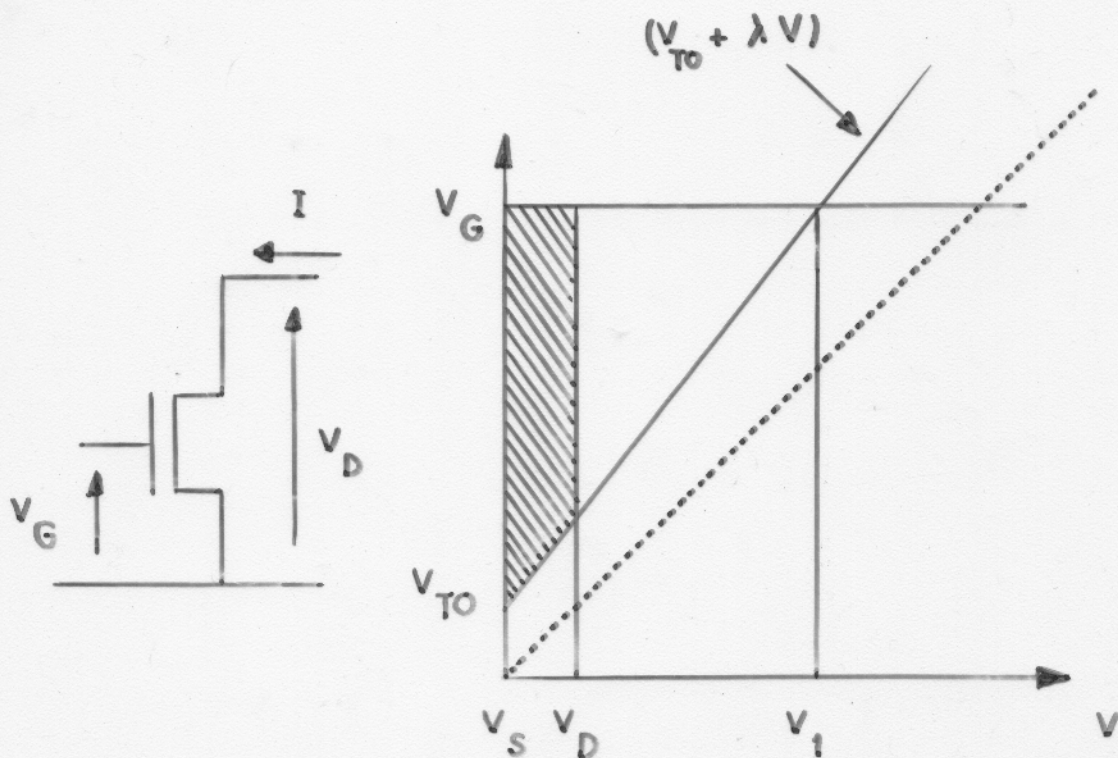
Model to analyze MOS transistor circuits

Let us replace equation (2) by:

$$V_T = V_{T0} + \lambda V \quad (3)$$

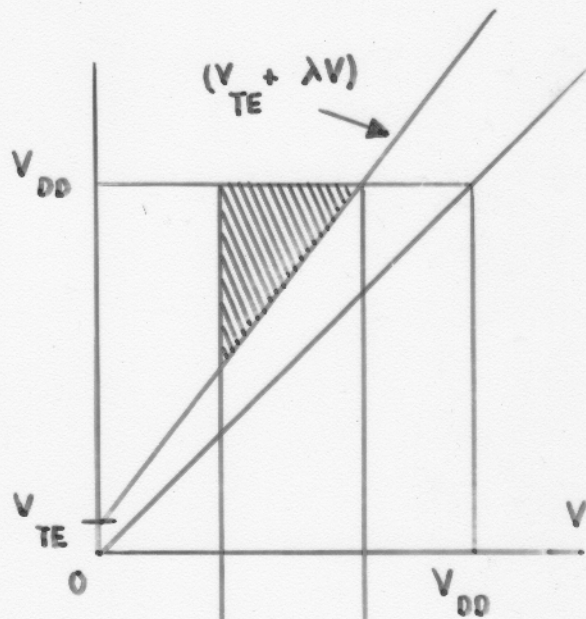
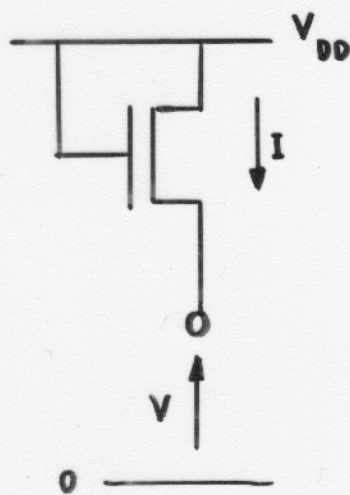
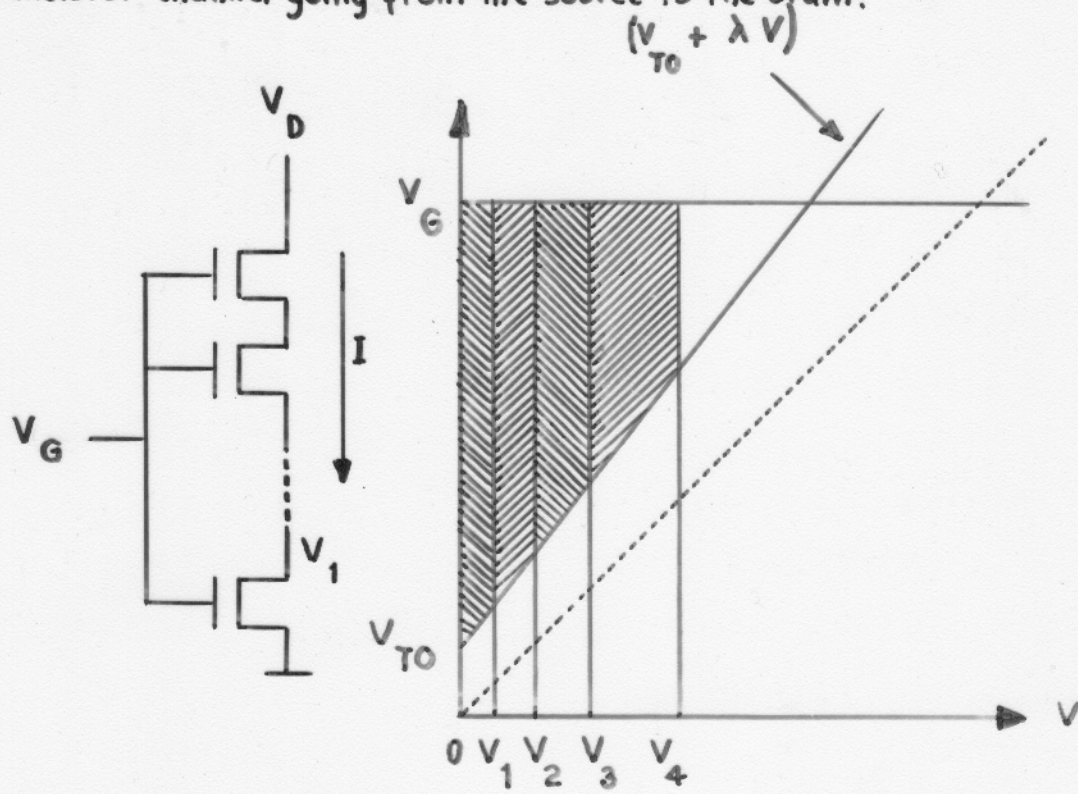


Graph representing the current in an MOS transistor.

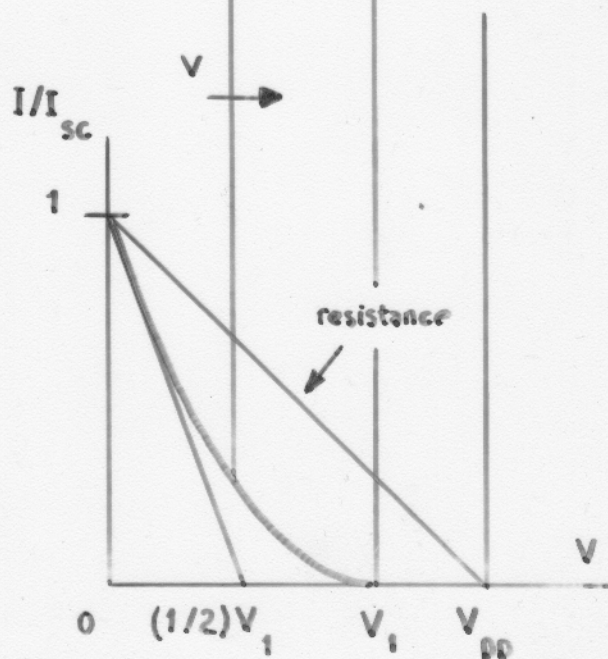


Graphical construction of the $I(V_G, V_D)$ characteristic

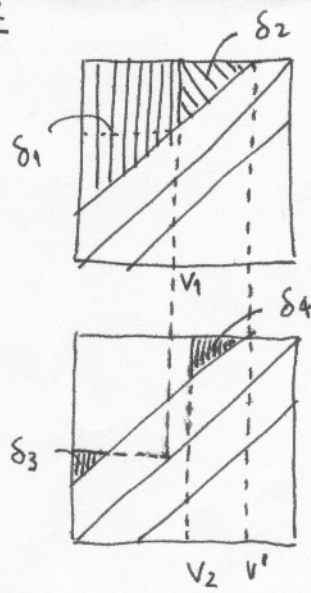
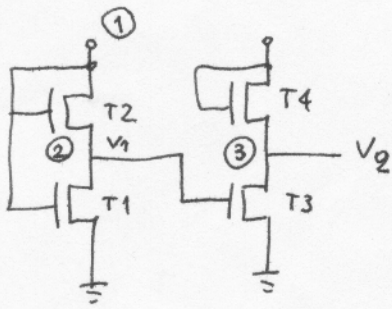
Graphical derivation of the quadratically increasing electrical field along the MOS transistor channel going from the source to the drain.



Comparison of the current flowing through an enhancement transistor load and a resistor.



nmos - ref



$$\delta_1 = \delta - \delta_2$$

$$\delta = \frac{\lambda V'^2}{2}$$

$$\delta_1 = \frac{\lambda V'^2}{2} - \frac{\lambda (V' - V_1)^2}{2}$$

$$\delta_2 = \frac{\lambda (V' - V_1)^2}{2}$$

$$V_{DD} = V_{T0} + \lambda V'$$

$$V' = \frac{V_{DD} - V_{T0}}{\lambda}$$

$$\left(\frac{W}{L}\right)_1 \delta_1 = \left(\frac{W}{L}\right)_2 \delta_2$$

$$\beta_1 = \frac{(W/L)_1}{(W/L)_2}$$

$$\beta_1 \left(\frac{\lambda V'^2}{2} - \frac{\lambda (V' - V_1)^2}{2} \right) = \frac{\lambda (V' - V_1)^2}{2}$$

$$\beta_1 V'^2 = (V' - V_1)^2 (1 + \beta_1)$$

$$\sqrt{\frac{\beta_1}{1+\beta_1}} V' = V' - V_1$$

$$V_1 = V' \left(1 - \sqrt{\frac{\beta_1}{1+\beta_1}} \right) //$$

$$\delta_3 = \frac{(V_1 - V_{T0})^2}{2\lambda}$$

$$(W/L)_3 \delta_3 = (W/L)_4 \delta_4$$

$$\delta_4 = \frac{\lambda (V' - V_2)^2}{2}$$

$$\beta_2 = \frac{(W/L)_3}{(W/L)_4}$$

$$\beta_2 \frac{(V_1 - V_{T0})^2}{2\lambda} = \frac{\lambda (V' - V_2)^2}{2}$$

$$\sqrt{\beta_2} (V_1 - V_{T0}) = \lambda (V' - V_2)$$

$$- \sqrt{\beta_2} V' \left(1 - \sqrt{\frac{\beta_1}{1+\beta_1}} \right) + \sqrt{\beta_2} V_{T0} + \lambda V' = + \lambda V_2$$

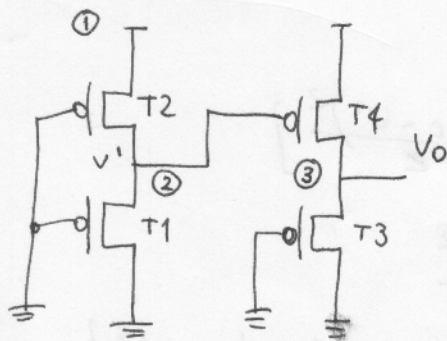
$$V_2 = \frac{V'}{\lambda} \left[\lambda - \sqrt{\beta_2} \left(1 - \sqrt{\frac{\beta_1}{1+\beta_1}} \right) \right] + \frac{\sqrt{\beta_2} V_{T0}}{\lambda}$$

$$V_2 = V' \left[1 - \frac{\sqrt{\beta_2}}{\lambda} \left(1 - \sqrt{\frac{\beta_1}{1+\beta_1}} \right) \right] + \frac{\sqrt{\beta_2} V_{T0}}{\lambda}$$

$$\text{Si } \frac{\sqrt{\beta_2}}{\lambda} = \frac{1}{1 - \sqrt{\frac{\beta_1}{1+\beta_1}}}$$

$\Rightarrow$

$$V_2 = \frac{\sqrt{\beta_2} V_{T0}}{\lambda} //$$



$$\delta_1 = \frac{(V' - V_{TP})^2}{2}$$

$$\delta = \frac{(V_{DD} - V_{TP})^2}{2}$$

$$\delta_2 = \delta - \delta_1$$

$$\mu_p C_{ox} \left(\frac{W}{L}\right)_1 \delta_1 = \mu_p C_{ox} \left(\frac{W}{L}\right)_2 \delta_2$$

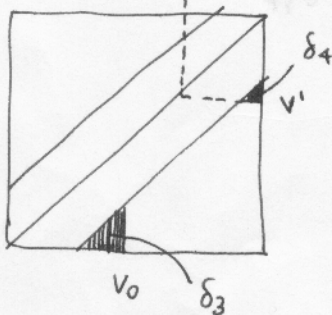
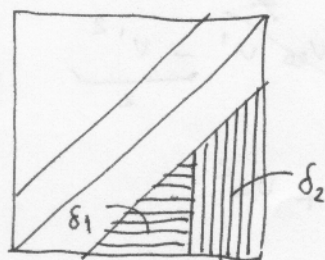
$$\frac{(W/L)_1}{(W/L)_2} = \beta_1 \quad \beta_1 \delta_1 = \delta_2$$

$$\beta_1 \frac{(V' - V_{TP})^2}{2} = \frac{(V_{DD} - V_{TP})^2}{2} - \frac{(V' - V_{TP})^2}{2}$$

$$\sqrt{(1 + \beta_1)} (V' - V_{TP}) = (V_{DD} - V_{TP})$$

$$V' = \frac{(V_{DD} - V_{TP})}{\sqrt{1 + \beta_1}} + V_{TP}$$

$$V' = \frac{V_{DD}}{\sqrt{1 + \beta_1}} + V_{TP} \left(1 - \frac{1}{\sqrt{1 + \beta_1}}\right)$$



$$\delta_4 = \frac{(V_{DD} - V_{TP} - V')^2}{2}$$

$$\delta_3 = \frac{(V_O - V_{TP})^2}{2}$$

$$\mu_p C_{ox} \left(\frac{W}{L}\right)_3 \delta_3 = \mu_p C_{ox} \left(\frac{W}{L}\right)_4 \delta_4$$

$$\frac{(W/L)_3}{(W/L)_4} = \beta_2 \quad \beta_2 \delta_3 = \delta_4$$

$$\sqrt{\beta_2} \frac{(V_O - V_{TP})^2}{2} = \frac{(V_{DD} - V_{TP} - V')^2}{2}$$

$$V_{DD} - V_{TP} - \frac{V_{DD}}{\sqrt{1 + \beta_1}} - V_{TP} \left(1 - \frac{1}{\sqrt{1 + \beta_1}}\right) + \sqrt{\beta_2} V_{TP} = \sqrt{\beta_2} V_O$$

$$\frac{V_{DD}}{\sqrt{\beta_2}} \left(1 - \frac{1}{\sqrt{1 + \beta_1}}\right) - \frac{V_{TP}}{\sqrt{\beta_2}} \left(1 + 1 - \frac{1}{\sqrt{1 + \beta_1}} - \sqrt{\beta_2}\right) = V_O$$

$$S_i \frac{\left(1 - \frac{1}{\sqrt{1 + \beta_1}}\right)}{\sqrt{\beta_2}} = 1$$

$\Rightarrow$

$$V_O = V_{DD} - \frac{V_{TP}}{\sqrt{\beta_2}}$$

$$I_D = \mu_p C_{ox} \left(\frac{W}{L} \right)_P \left[(V_{GS} - V_{TP}) V_{DS} - \frac{V_{DS}^2}{2} \right]$$

$$= \mu_p C_{ox} \left(\frac{W}{L} \right)_P \left[(V_{DD} - V_{TP}) (V_{DD} - V') - \frac{(V_{DD} - V')^2}{2} \right]$$

$$\beta_1 \frac{(V' - V_{TP})^2}{2} = (V_{DD} - V_{TP}) (V_{DD} - V') - \frac{(V_{DD} - V')^2}{2}$$

$$= \frac{2(V_{DD}^2 - V_{DD} V_{TP} + V' V_{TP}) - [V_{DD}^2 - 2 V_{DD} V' + V'^2]}{2}$$

$$\beta_1 (V' - V_{TP})^2 = \underbrace{V_{DD}^2 - 2 V_{DD} V_{TP}}_{\text{cancel}} + \underbrace{2 V' V_{TP}}_{\text{cancel}} + \cancel{2 V_{DD} V'} - \frac{V'^2}{2}$$

$$(V_{DD} - V_{TP})^2 = V_{DD}^2 - 2 V_{DD} V_{TP} + V_{TP}^2$$

$$-(V' - V_{TP})^2 = -V'^2 + 2 V' V_{TP} - V_{TP}^2$$

$$(V_{DD} - V_{TP})^2 - \frac{V_{TP}^2}{2} = (V' - V_{TP})^2$$

$$\beta_1 (V' - V_{TP})^2 = (V_{DD} - V_{TP})^2 - (V' - V_{TP})^2$$