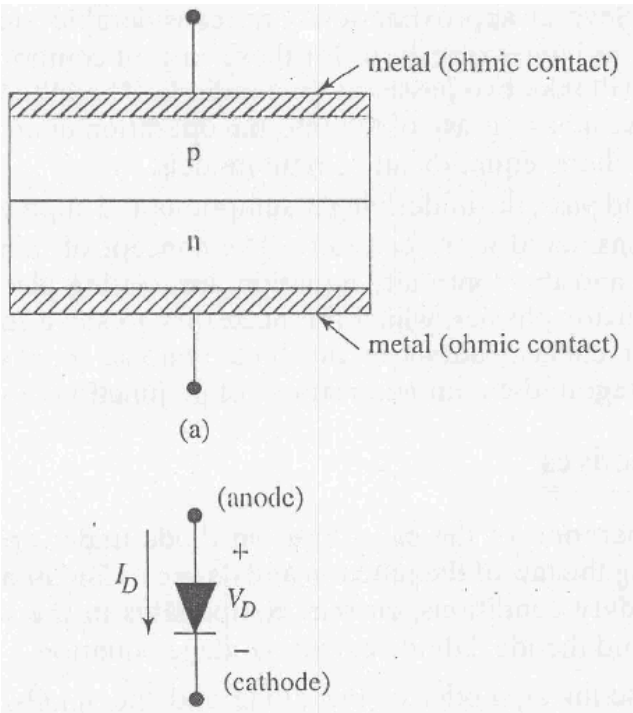
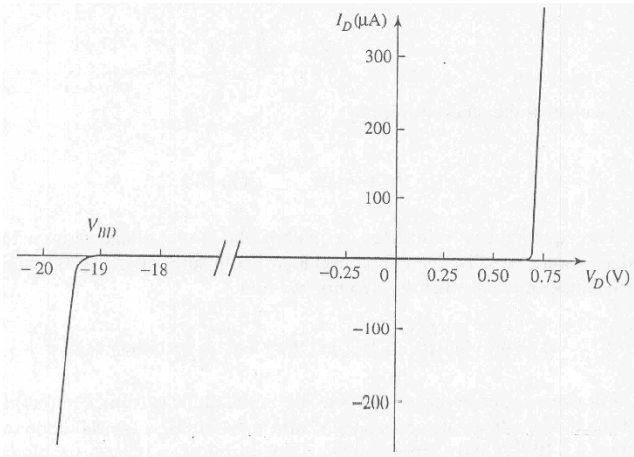
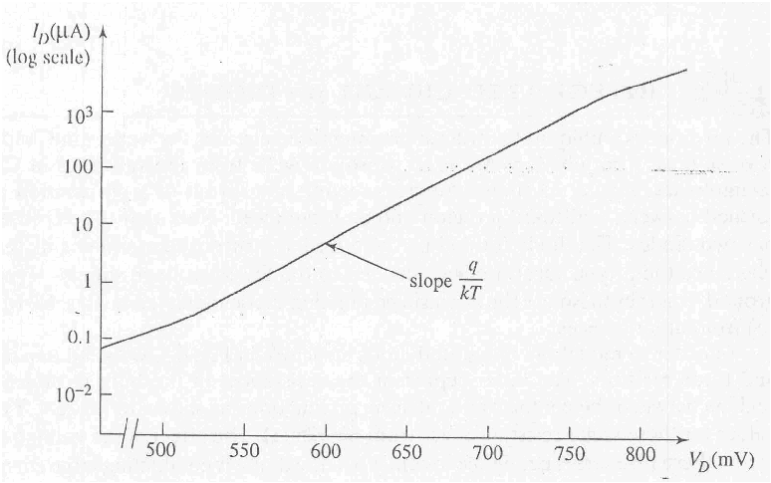
Diode Circuit Symbol and Characteristics







$$I_D = I_o \cdot e^{qV_D/kT} \quad (V_D > 0)$$

$$I_D = I_o \cdot e^{V_D/V_{th}} \quad (V_D > 0)$$

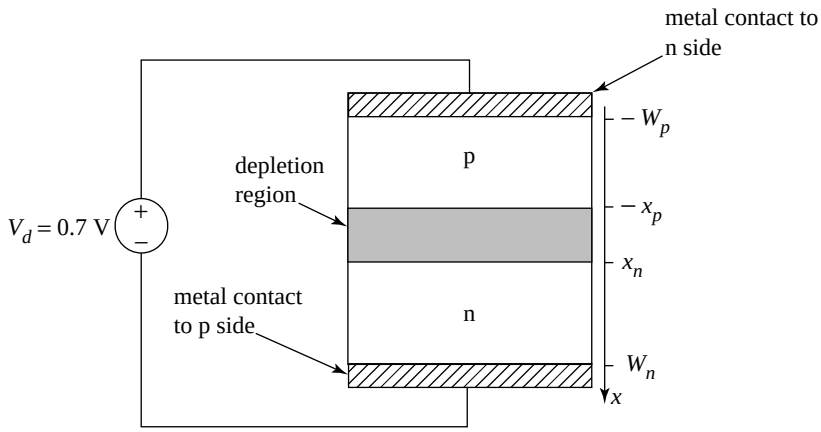
Io: Diode saturation current

$$V_D = V_{th} \cdot \ln\left(\frac{I_D}{I_o}\right) = V_{th} \cdot \ln(10) \cdot \log\left(\frac{I_D}{I_o}\right) = 60\text{mV} \log\left(\frac{I_D}{I_o}\right)$$

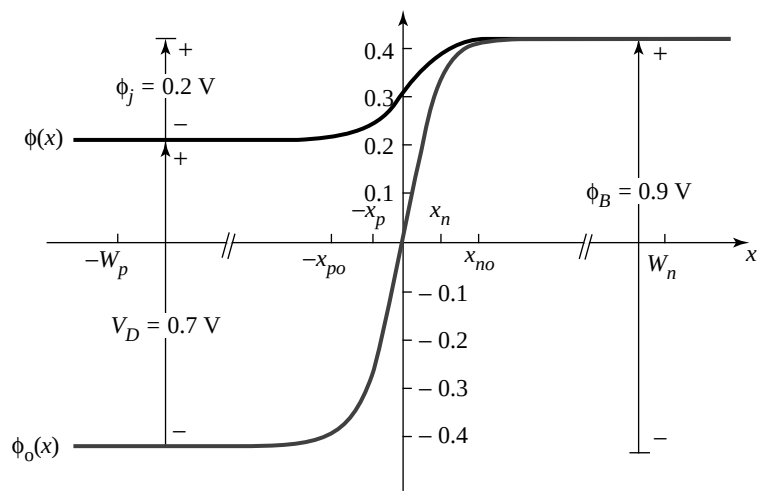
The pn Junction under Forward Bias

- $V_D > 0$ --> what happens?

  Many assumptions: from Chapter 6 (current not too big) --> resistive potential drops in bulk p & n regions can be neglected in KVL and $\phi_j = \phi_B - V_D$



(a)



(b)

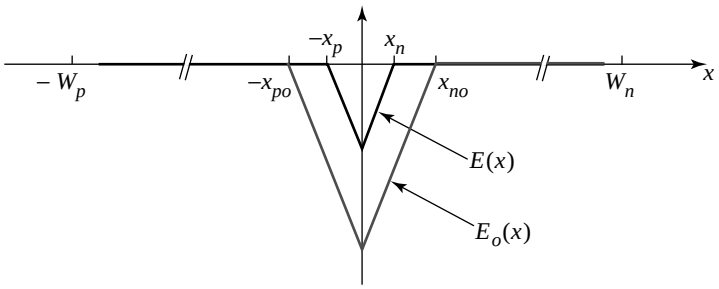
ϕ_B = thermal equilibrium barrier height = $\phi_n - \phi_p$

Physical Reasoning

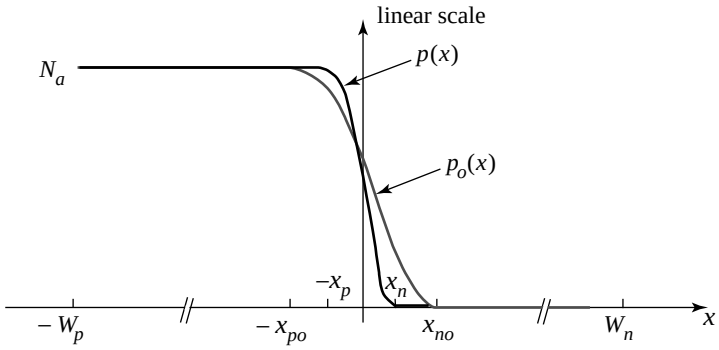
- thermal equilibrium --> balance between drift and diffusion:

  $J = J^{drift} + J^{diff} = 0$ for holes and electrons

- forward bias upsets balance



(a)



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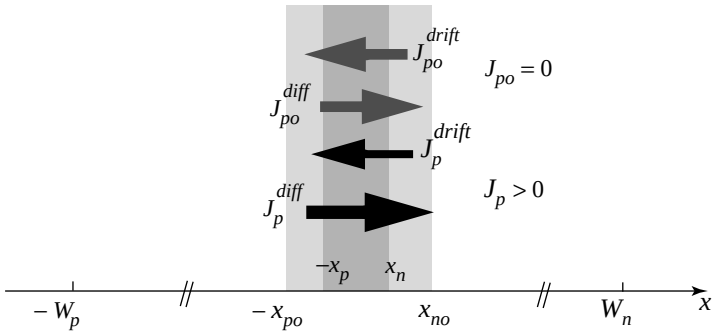


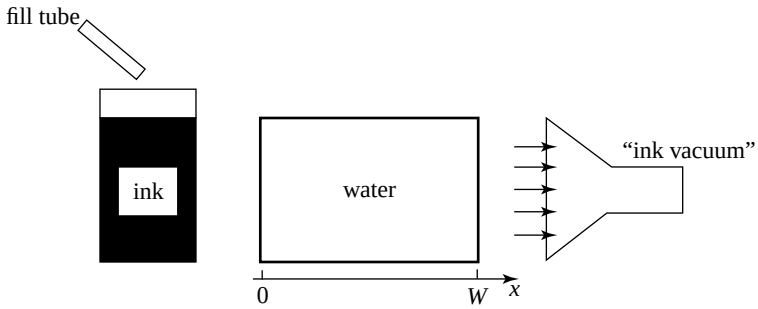
(c)

Diffusion Transport in Steady-State

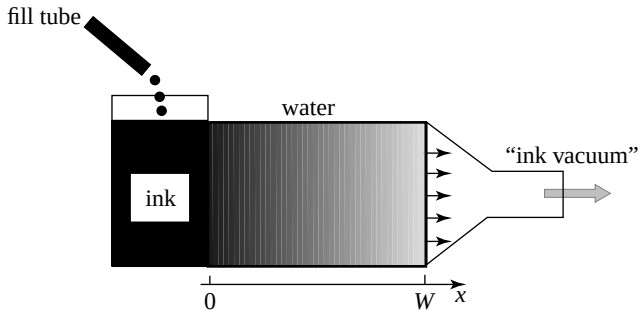
- How do we “fill in the blanks” between the contacts and the depletion region?

  Steady-state --> minority carriers must be continuously injected across the junction to keep $p_n(x_n) >> p_{no}$ and $n_p(-x_p) >> n_{po}$ and continously extracted at the contacts; huge gradient in minority carrier concentrations across the n and p regions --> transport occurs by *diffusion*.

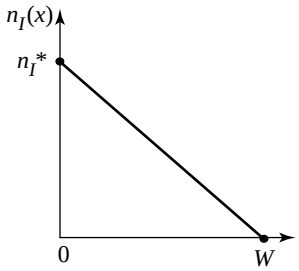
- Conceptual experiment:



(a)



(b)



(c)

Modelling Forward-Bias Diode Currents

- **Step 1**: find how minority carrier concentrations at the edges of depletion region change with forward bias V_D

- **Step 2**: what happens to the minority carrier concentration at the ohmic contacts under forward bias? *Answer*: no change from equilibrium.

- **Step 3**: find the minority carrier concentrations $n_p(x)$ in the p region and $p_n(x)$ in the n region.

- **Step 4**: find the minority carrier diffusion currents.

- **Step 5**: find the total current J.

Carrier Concentrations in Thermal Equilibrium at the pn Junction

- For the junction in thermal equilibrium,

$$\phi_B = \frac{kT}{q} ln(\frac{N_a N_d}{n_i^2}), \text{ where}$$

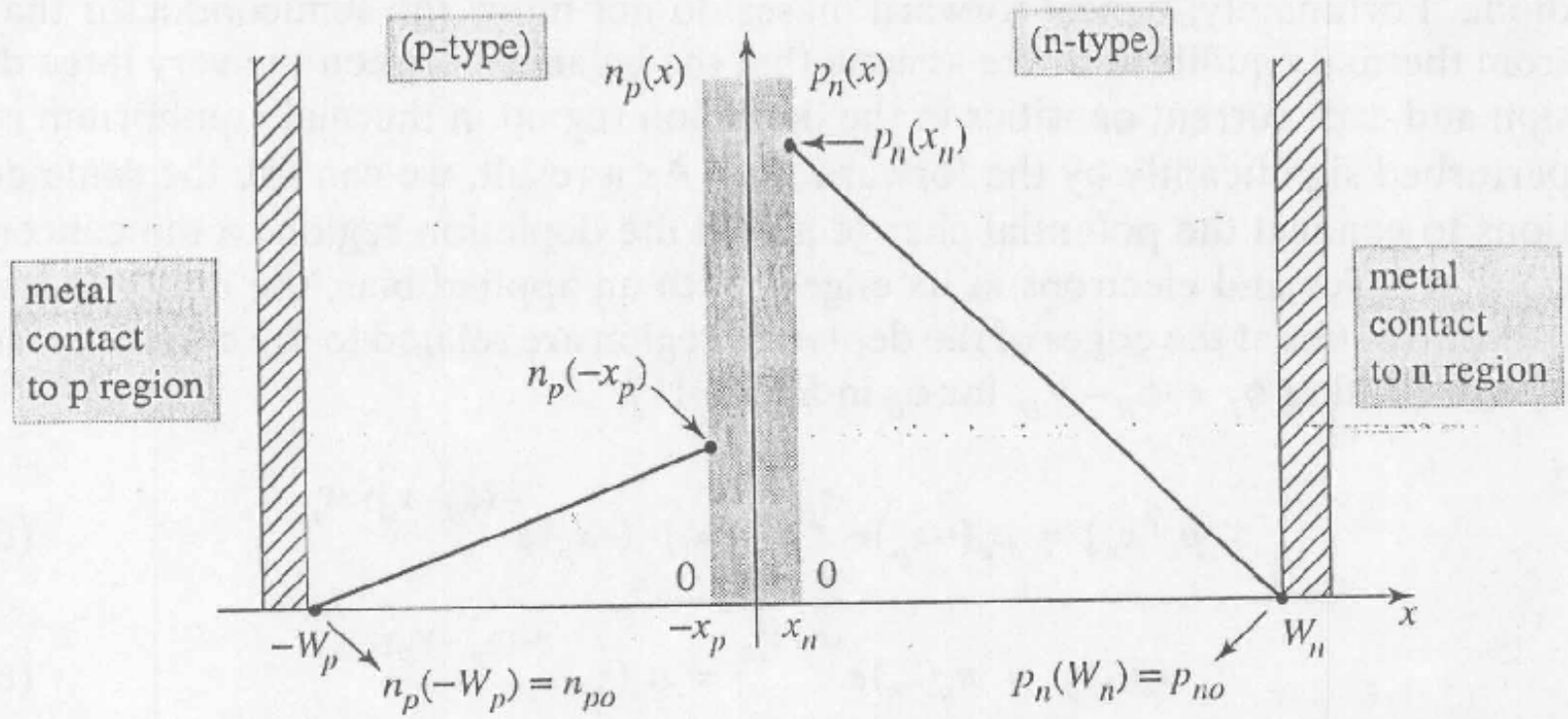
If we identify $p_{no} = n_i^2 / N_d$ and $n_{po} = n_i^2 / N_a$, we can reexpress this basic result in two ways --

$$\phi_B = V_{th} ln(\frac{N_d}{n_{po}}) \qquad \text{and} \qquad \phi_B = V_{th} ln(\frac{N_a}{p_{no}}).$$

- Solving for the equilibrium minority carrier concentrations in terms of the built-in potential,

$$p_{no} = N_a e^{-\phi_B/(V_{th})} \qquad \text{and} \qquad n_{po} = N_d e^{-\phi_B/(V_{th})} \quad .$$

This result is very important, since it relates the minority carrier concentration on one side of the junction to the majority carrier concentration on the *other side* of the junction ... !



➤ **Figure 6.8** Minority carrier concentrations on a linear scale for a forward-biased short-base diode.

Law of the Junction

- What happens under an applied bias?

  assume that the new potential barrier $\phi_j = \phi_B - V_D$ can substituted for the thermal equilibrium barrier to find the new minority carrier concentrations *at the depletion region edges* $-x_p$ (p-side) and x_n (n-side)

$$n_p(-x_p) = N_d e^{-\phi_j/V_{th}} = N_d e^{-(\phi_B - V_D)/V_{th}} \text{ and}$$

$$p_n(x_n) = N_a e^{-\phi_j/V_{th}} = N_a e^{-(\phi_B - V_D)/V_{th}}.$$

These results can be re-expressed in a simpler form, by expanding the exponentials:

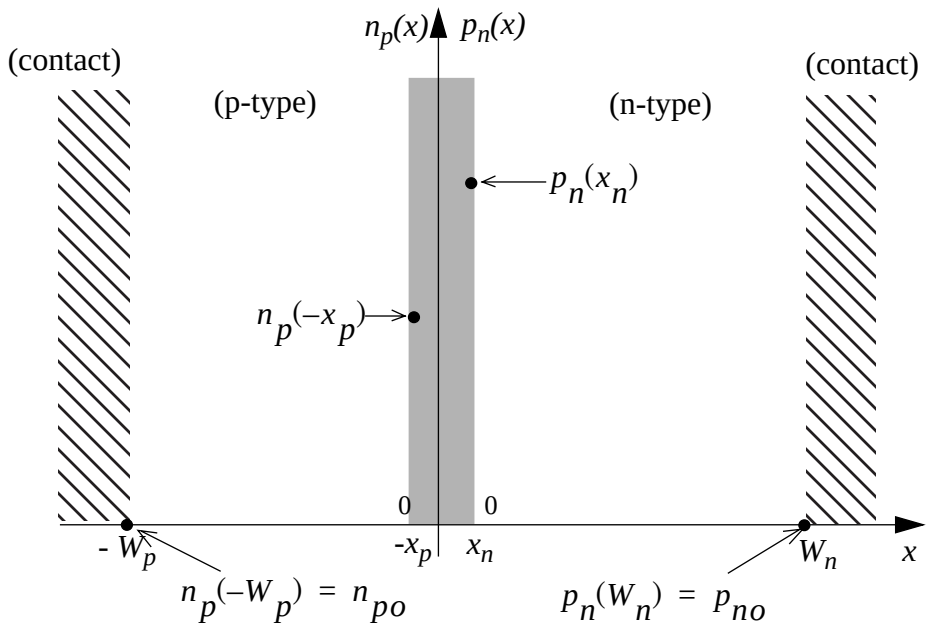
$$\boxed{\begin{aligned} n_p(-x_p) &= N_d e^{-\phi_B/V_{th}} e^{V_D/V_{th}} = n_{po} e^{V_D/V_{th}} \\ p_n(x_n) &= N_a e^{-\phi_B/V_{th}} e^{V_D/V_{th}} = p_{no} e^{V_D/V_{th}} \end{aligned}}$$

- These two equations are known as the **Law of the Junction**.

  Note that the minority carrier concentration is an exponential function of the applied bias on the junction.

Carrier Concentrations under Forward Bias

- Apply the Law of the Junction at the edges of the depletion region



- Numerical values: $N_a = 10^{18}$ cm^{-3}, $N_d = 10^{17}$ cm^{-3}

  V_D = 0.72 V = 720 mV, V_{th} = 26 mV (warm room) ... example values; note that V_D must be known precisely to substitute into exp[V_D/V_{th}].

$$n_p(-x_p) = 10^2 \text{ cm}^{-3} \exp[720/26] = 10^{14} \text{ cm}^{-3}$$

$$p_n(x_n) = 10^3 \text{ cm}^{-3} \exp[720/26] = 10^{15} \text{ cm}^{-3}$$

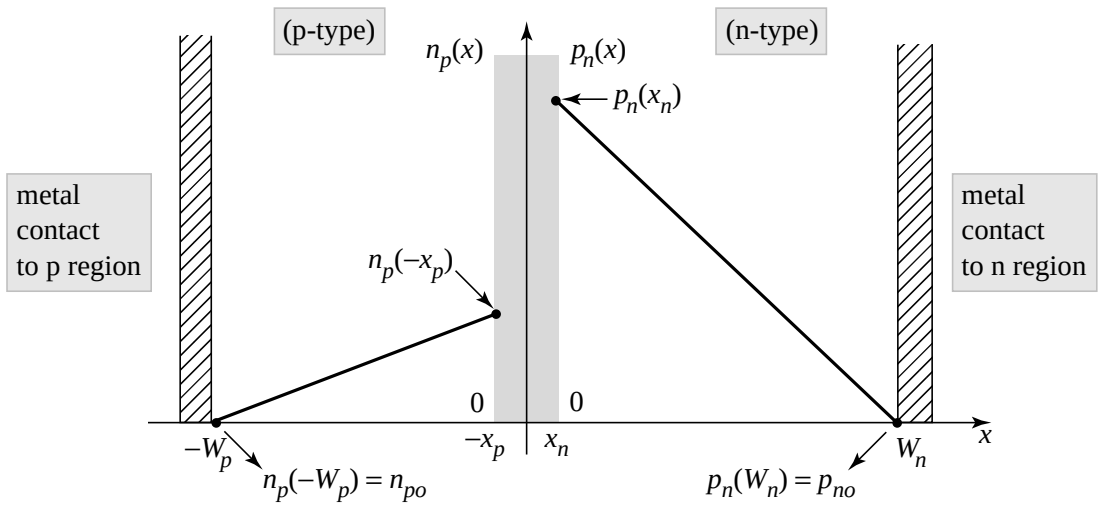
- The minority carrier concentration is maintained at thermal equilibrium at the ohmic contacts

The Short-Base Solution

- Carrier concentrations --> linear solutions if we assume that the p-type and n-type regions are so short that all of the diffusing minority carriers "make it" across to the ohmic contacts

  In n-type region: $J_p^{diff} = -\, qD_p dp_n / dx$ = constant --> $p_n(x)$ is *linear*

  In p-type region: $J_n^{diff} = qD_n dn_p / dx$ = constant --> $n_p(x)$ is *linear*



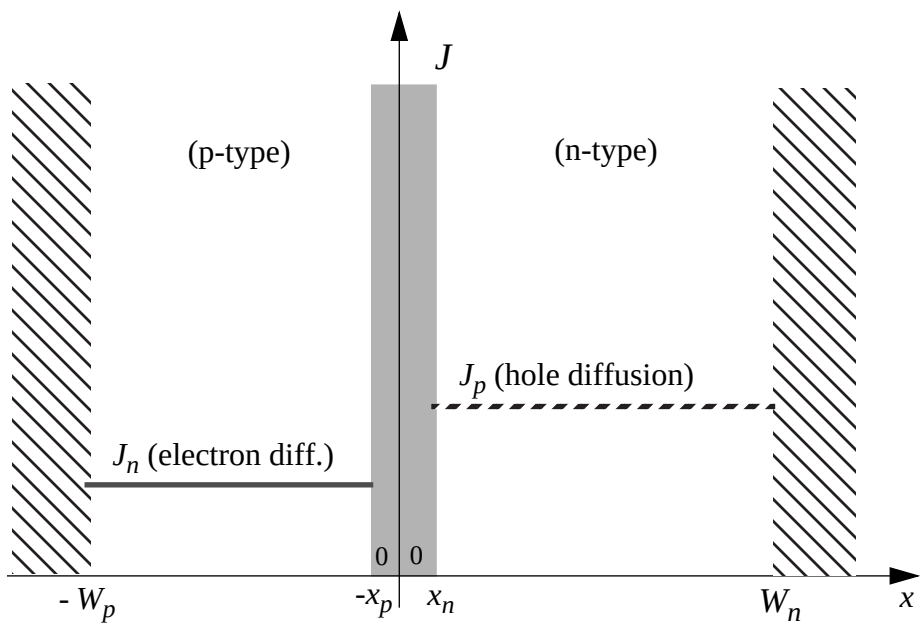
Current Densities

- Minority carrier diffusion currents

$$J_n^{diff} = qD_n \frac{dn_p}{dx} = qD_n \left(\frac{n_p(-x_p) - n_p(-W_p)}{W_p - x_p} \right) = qD_n n_{po} e^{V_D/V_{th}}$$

$$J_p^{diff} = -qD_p \frac{dp_n}{dx} = -qD_p \left(\frac{p_n(W_n) - p_n(x_n)}{W_n - x_n} \right) = qD_p p_{no} e^{V_D/V_{th}}$$

- Plot of minority carrier diffusion current densities

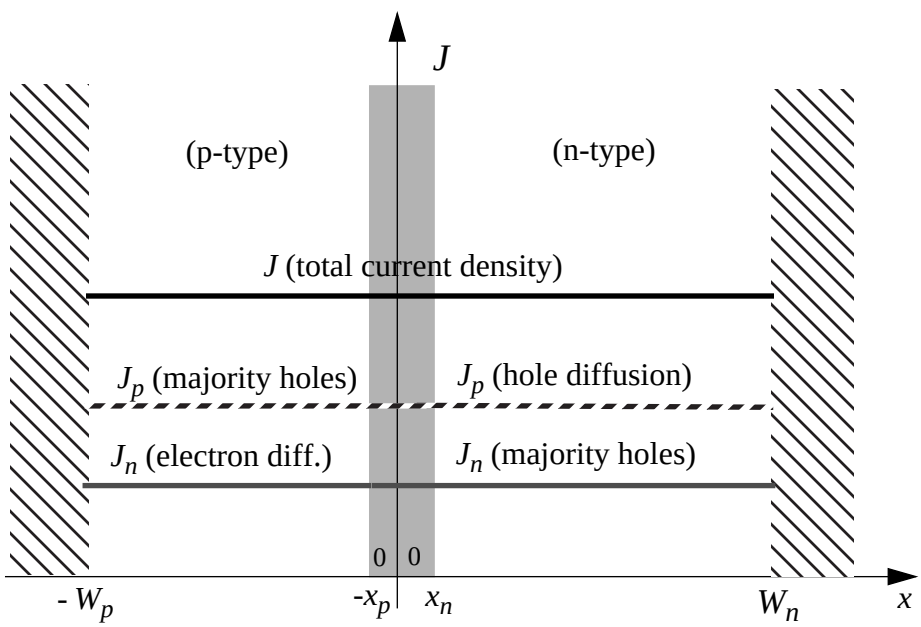


- Minority carriers are injected from the other side of the junction ... how do they get there? *by a majority carrier current density*

Total Current Density

- The total current density is the sum of the minority electron and hole diffusion current densities at the junction ... and is constant through the diode

$$J = J_n^{diff} + J_p^{diff}$$



$$J = q\left(\frac{D_n n_{po}}{W_p - x_p} + \frac{D_p p_{no}}{W_n - x_n}\right)(e^{V_D/V_{th}} - 1)$$

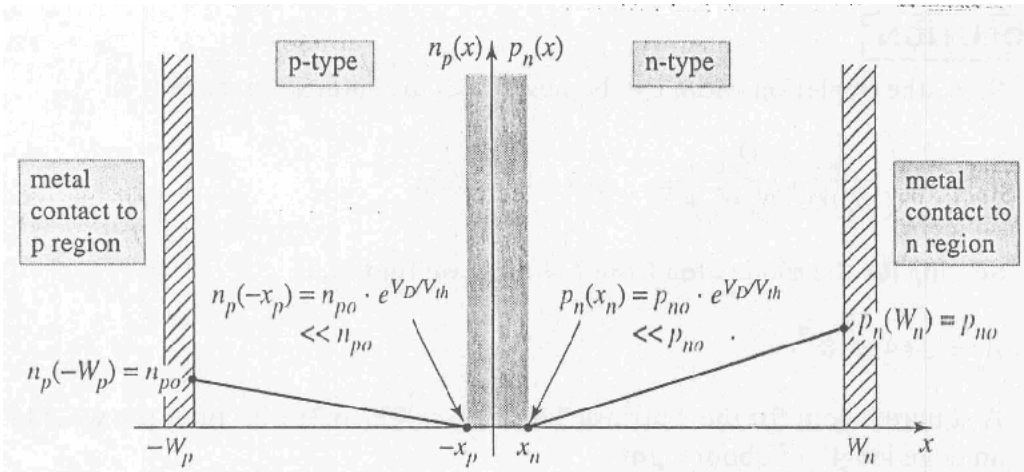
- Diode current: multiply by area A and note that x_n, $x_p << W_n$, W_p

$$I = qn_i^2\left(\frac{D_n}{N_a W_p} + \frac{D_p}{N_d W_n}\right)(e^{V_D/V_{th}} - 1) = I_o(e^{V_D/V_{th}} - 1)$$

Diode under reverse bias

- Under a reverse bias >0.1mV, minority carriers become negligible
- `

$$p_n(x_n) = p_{no} e^{V_D/V_{th}} \ll p_{no} \text{ and } n_p(-x_p) = n_{po} e^{V_D/V_{th}} \ll n_{po} \; (V_D < -0.1 \text{ V})$$

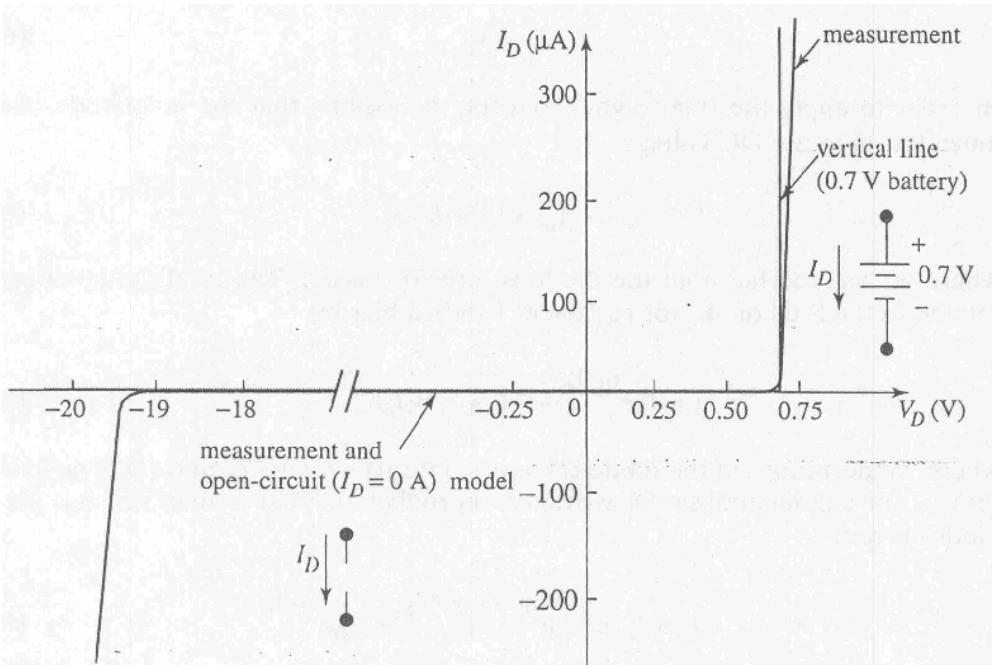


$$J \approx -qD_p\left[\frac{p_n(W_n) - 0}{W_n - x_n}\right] + qD_n\left(\frac{0 - n_p(-W_p)}{W_p - x_p}\right) \approx -q\left(\frac{D_p p_{no}}{W_n} + \frac{D_n n_{po}}{W_p}\right)$$

- Minority carriers are extracted
- This simple model predicts constant current. It actually increases with reverse bias, and is much larger than Io.

P-N junction diode circuit models

- Large signal static model



P-N junction diode circuit models

- Small signal model

$$i_D = I_O e^{v_D/V_{th}} \rightarrow I_D + i_d = I_O e^{(V_D + v_d)/V_{th}}$$

$$I_D + i_d = I_D\left(1 + \frac{v_d}{V_{th}} + \frac{1}{2}\left(\frac{v_d}{V_{th}}\right)^2 + \frac{1}{6}\left(\frac{v_d}{V_{th}}\right)^3 + \ldots\right)$$

$$i_d = I_D\left(\frac{v_d}{V_{th}}\right) + \frac{I_D}{2}\left(\frac{v_d}{V_{th}}\right)^2 + \frac{I_D}{6}\left(\frac{v_d}{V_{th}}\right)^3 + \ldots$$

$$i_d \cong \left(\frac{I_D}{V_{th}}\right) v_d = g_d v_d$$

$$r_d = \frac{1}{g_d} = \frac{V_{th}}{I_D}$$

P-N junction diode circuit models

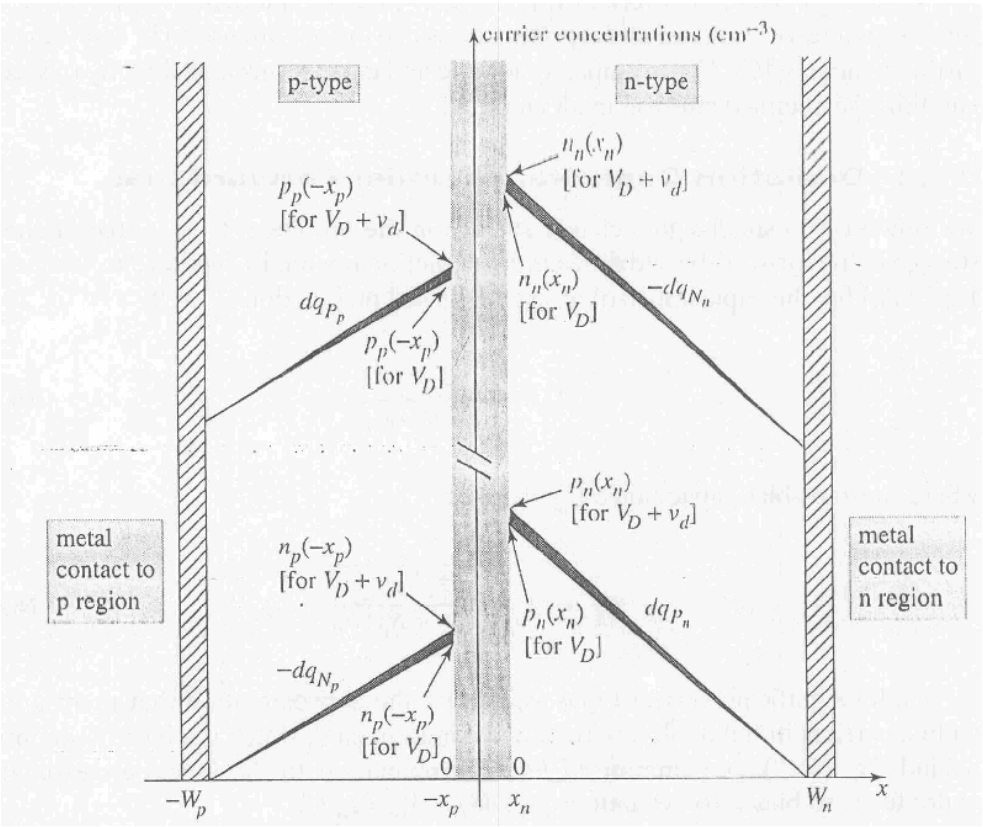
- Depletion Capacitance
  - With reverse bias

$$C_j = \frac{C_{jo}}{\sqrt{1 - V_D/\phi_B}}$$

$$C_{jo} = A\sqrt{\frac{q\varepsilon_s N_a N_d}{2(N_a + N_d)\phi_B}}$$

Under significant forward bias $V_D > \phi_B / 2$ the approximation that there are no mobile carriers in the depletion region becomes invalid, which is a basic assumption behind Eq. (6.48). A common *ad hoc* approximation to the junction capacitance under forward bias is to evaluate Eq. (6.48) at $V_D = \phi_B / 2$

$$\boxed{C_j \cong \frac{C_{jo}}{\sqrt{1 - 1/2}} = \sqrt{2}C_{jo}} \text{ for forward bias}$$

P-N junction diode circuit models

- Diffusion Capacitance