Electrostatics:
The Key to Understanding Electronic Devices

Physics approach: vector calculus, highly symmetrical problems

Gauss's Law: $\nabla \bullet (\varepsilon \vec{E}) = \rho$

Def. of Potential: $\vec{E} = -\nabla\phi$

Poisson's Eqn.: $\nabla \bullet (\varepsilon(-\nabla\phi)) = \varepsilon\nabla^2\phi = \rho$

Device physics

Real problems (not symmetrical, complicated boundary conditions)

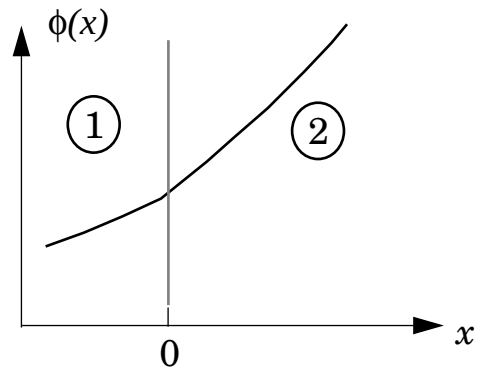
Gauss's Law: $\dfrac{d(\varepsilon E)}{dx} = \rho$

Definition of Potential: $E = -\dfrac{d\phi}{dx}$

Poisson's Equation: $\dfrac{d}{dx}\left(\varepsilon\left(-\dfrac{d\phi}{dx}\right)\right) = -\varepsilon\dfrac{d^2\phi}{dx^2} = \rho$

Boundary Conditions

1. Potential: $\phi(x=0^-) = \phi(x=0^+)$



2. Electric Field: $\varepsilon_1 E(x=0^-) + Q = \varepsilon_2 E(x=0^+)$

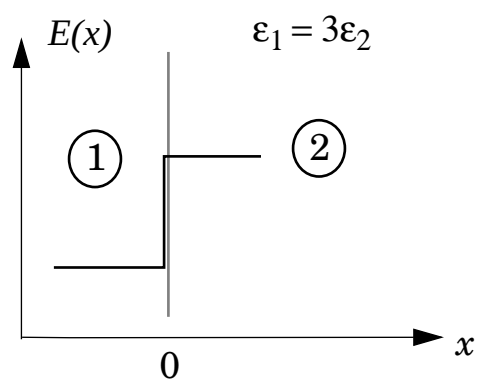
where Q is a surface charge (units, C/cm^2) located at the interface

for the case where $Q = 0$:

common materials:

silicon, $\varepsilon_s = 11.7\ \varepsilon_o$

silicon dioxide (SiO_2), $\varepsilon_{ox} = 3.9\ \varepsilon_o$



Intuition for Electrostatics

"Rules of Thumb" for sketching the solution BEFORE doing the math:

- Electric field points from positive to negative charge
- Electric field points "downhill" on a plot of potential
- Electric field is confined to a narrow charged region, in which the positive charge is balanced by an equal and opposite negative charge
- Use boundary conditions on potential or electric field to "patch" together solutions from regions having different material properties
- Gauss's law in integral form relates the electric field at the edges of a region to the charge inside. Often, the field on one side is known to be zero (e.g., because it's on the outside of the charged region), which allows the electric field at an interface to be solved for directly

Charge density functions: only two cases needed for basic device physics

$\rho = 0$ —–> E constant --> ϕ linear

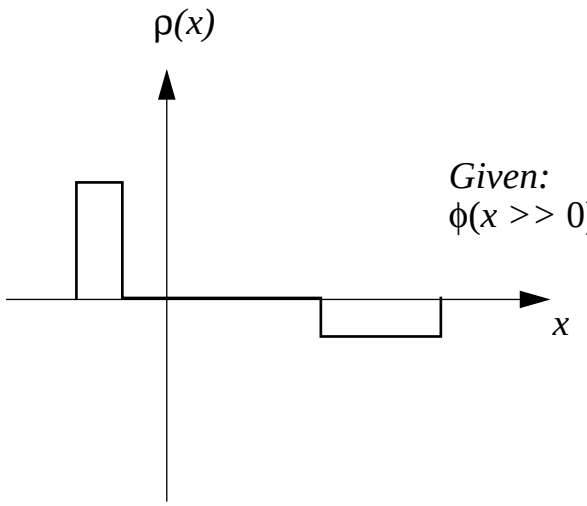
$\rho = \rho_o$ = constant --> E linear --> ϕ quadratic

- surface or sheet charge Q is sometimes present (e.g., on the surface of good conductors); the effect on electric field can be incorporated through the boundary condition

Example I: Applied Electrostatics

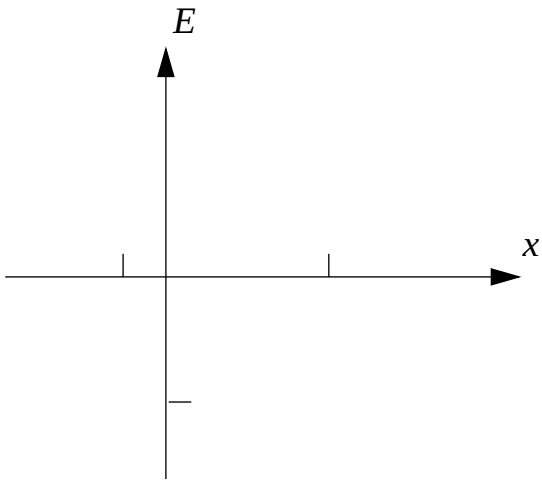
Given:
charge distribution
$\rho(x)$

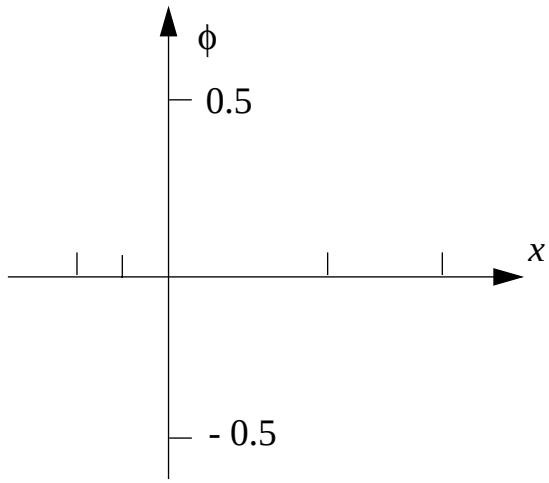
Given:
$\phi(x << 0) = 0.5$ V



Given:
$\phi(x >> 0) = -0.4$ V

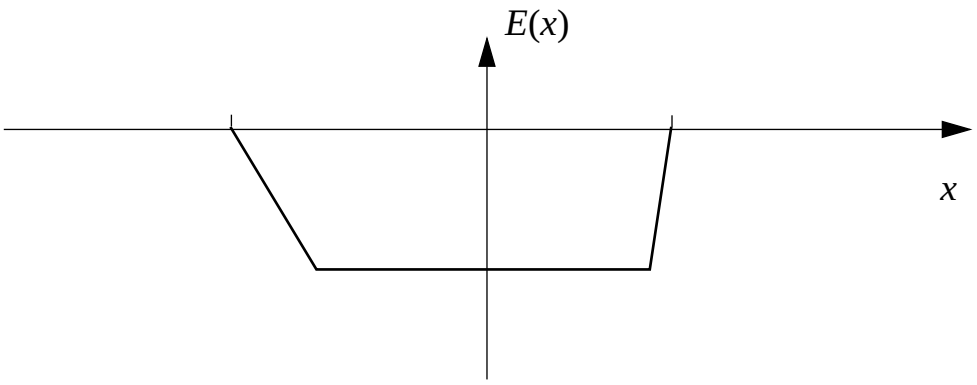
Sketch the electric field and the charge.



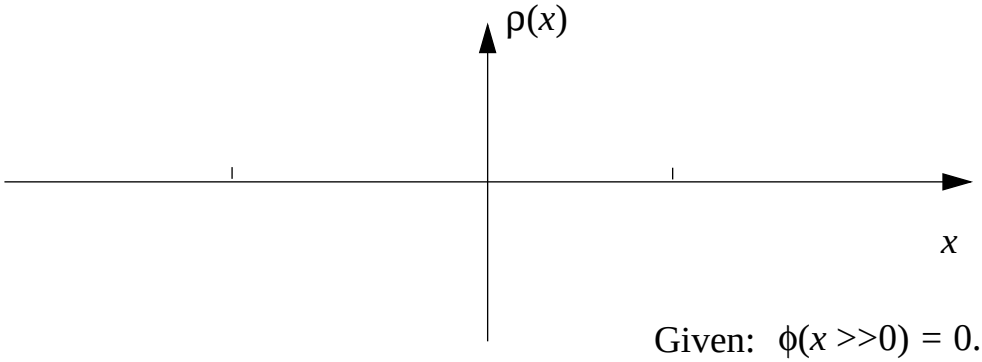


Example II: Applied Electrostatics

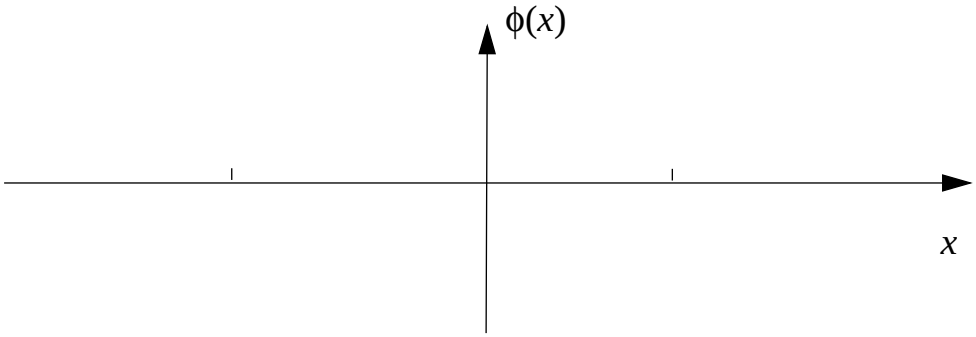
* Given the electric field,



Sketch the charge density and the potential



Given: $\phi(x >> 0) = 0.3$ V



Application of Gauss's Law

- At a point x, the electric field can be found as the charge enclosed, divided by the permittivity of the material ...

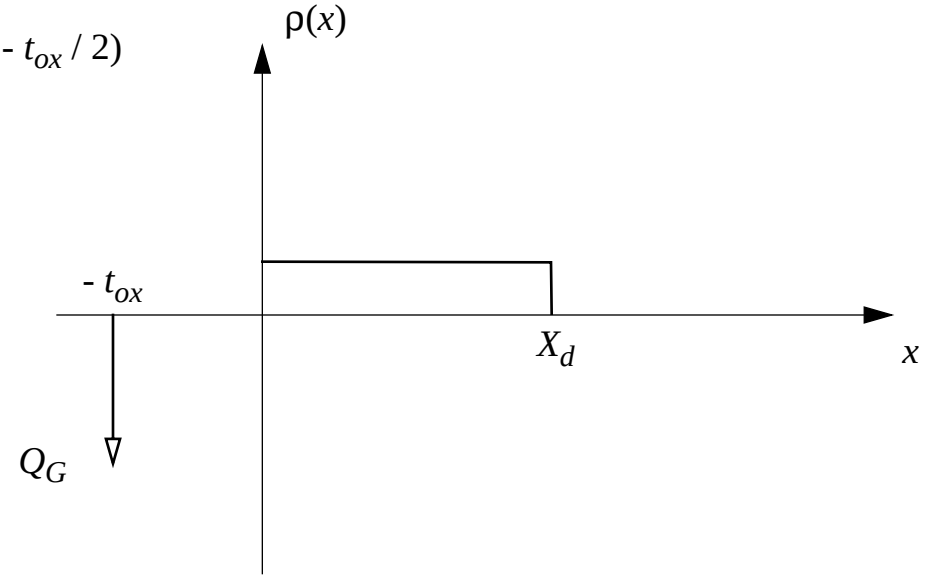
  caveats (warnings):

  (i) the field must be zero at the other side of the charged region

  (ii) the sign of the field can be found by keeping track of the $+x$ direction and the one-dimensional equivalent of the "outward normal;" however, the best approach is to know the sign of the field from the distribution of charge in the problem

- Example: metal-oxide-silicon structure

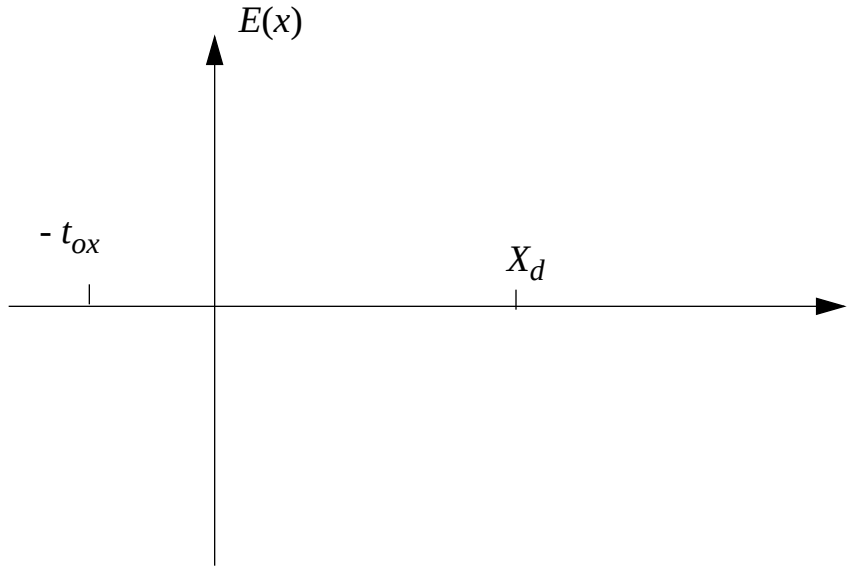
  Find $E(x = -t_{ox} / 2)$



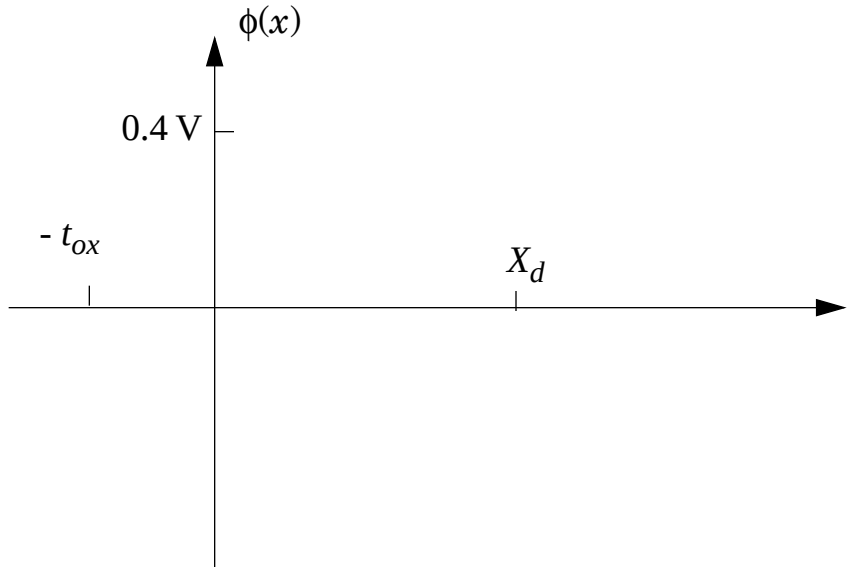
- Find $E(x = 0^+)$ ... just inside the silicon

Boundary Condition on *E* (cont.)

- Sketch $E(x)$ from $x = -t_{ox}$ to $x = X_d$



- Sketch $\phi(x)$ through the structure, given that $\phi(X_d) = 400$ mV



Potential and Carrier Concentration in Silicon

- What is a convenient reference for the electrostatic potential in silicon?

Thermal equilibrium: no external stimulus --> must have:

$$J_p = 0 \text{ and } J_n = 0.$$

$$\therefore 0 = qn_o\mu_n E + qD_n\frac{dn_o}{dx} = qn_o\mu_n\left(-\frac{d\phi}{dx}\right) + qD_n\frac{dn_o}{dx}$$

$$d\phi = \frac{D_n}{\mu_n}\left(\frac{dn_o}{n_o}\right) = \frac{kT}{q}\left(\frac{dn_o}{n_o}\right) = V_{th}\left(\frac{dn_o}{n_o}\right)$$

where we have used Einstein's relation.

$$\frac{D_n}{\mu_n} = \frac{kT}{q} = V_{th} = 25 \text{ mV at "cool" room temperature to}$$

26 mV at "warm" room temperature

V_{th} is called the *thermal voltage.*

The Intrinsic Potential Reference

- By integrating the equation relating potential to the electron concentration from a position x_a to position x, we find that:

$$\phi(x) - \phi(x_a) = V_{th}\ln\left(\frac{n_o(x)}{n_o(x_a)}\right)$$

We can choose any reference; one convenient choice is to set:

$\phi(x_a) = 0$ when $n_o(x_a) = n_i = 10^{10}$ cm^{-3} at room temperature.

- Using this reference, the potential in thermal equilibrium can be found, given the electron concentration:

$$\phi = V_{th}\ln\left(\frac{n_o}{n_i}\right) = (26\text{mV})\ln(10)\log\left(\frac{n_o}{10^{10}}\right) = (60\text{mV})\log\frac{n_o}{10^{10}}$$

Donor concentrations from 10^{13} to 10^{19} cm^{-3} therefore correspond to potentials of (60 mV) x 3 = 180 mV to (60 mV) x 9 = 540 mV (at room temperature)

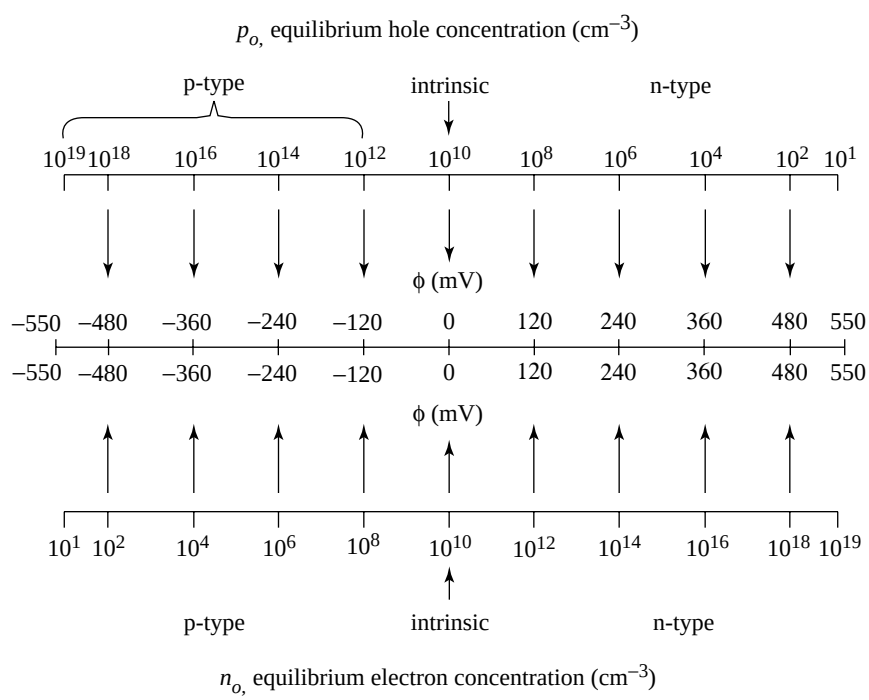
The 60 mV Rule

The hole concentration can also be related to the potential, by repeating the derivation starting with $J_p = 0$ or by substituting

$$p_o = n_i^2 / n_o$$

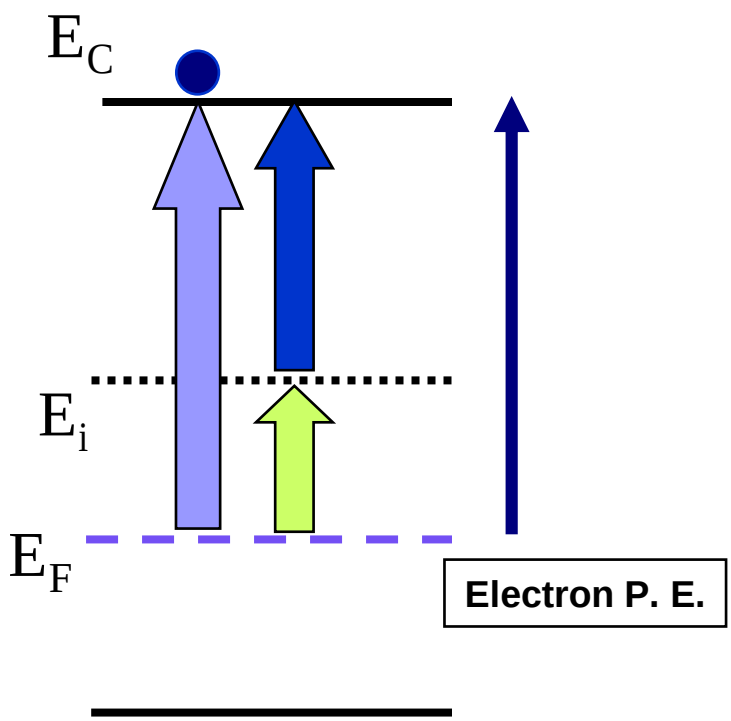
into the 60 mV rule for electrons. The result is:

$$\phi = V_{th}\ln\left(\frac{n_i}{p_o}\right) = (-26\text{mV})\ln(10)\log\left(\frac{p_o}{10^{10}}\right) = (-60\text{mV})\log\left(\frac{p_o}{10^{10}}\right)$$



Potential Energy (PE) of Electrons

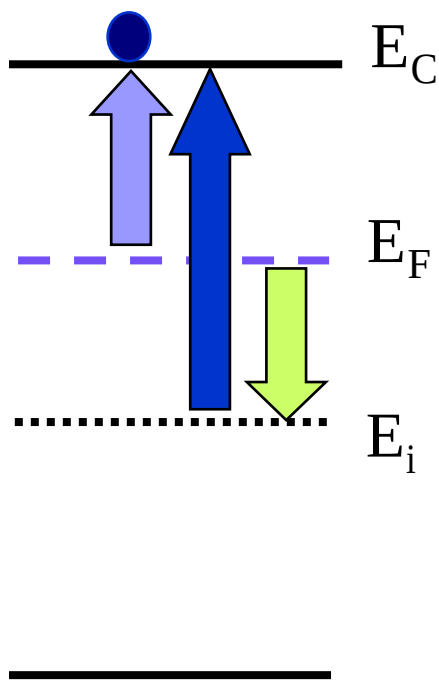
$$\phi = -\frac{1}{q}\left(E - E_{ref}\right) = -\frac{1}{q}\left(E_i - E_F\right)$$



$E = E_C - E_F$

$E = (E_C - E_i) + (E_i - E_F)$

$E = E_i - E_F$



$$\phi_p = -\frac{kT}{q}\ln\left(\frac{N_A}{n_i}\right)$$

$$\phi_n = \frac{kT}{q}\ln\left(\frac{N_D}{n_i}\right)$$

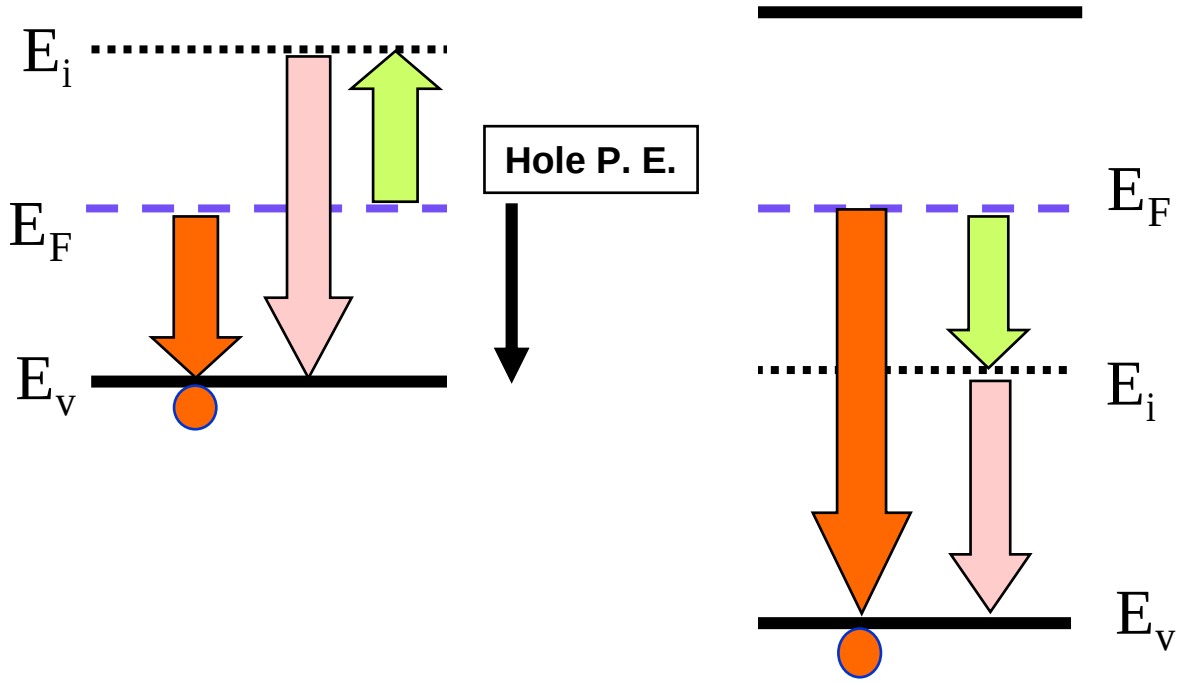
Potential Energy (PE) of Holes

$$\phi = \frac{1}{q}\left(E - E_{ref}\right) = \frac{1}{q}\left(E_i - E_F\right)$$

$$E = E_v - E_F$$

$$E = (E_v - E_i) + (E_i - E_F)$$

$$E = E_i - E_F$$

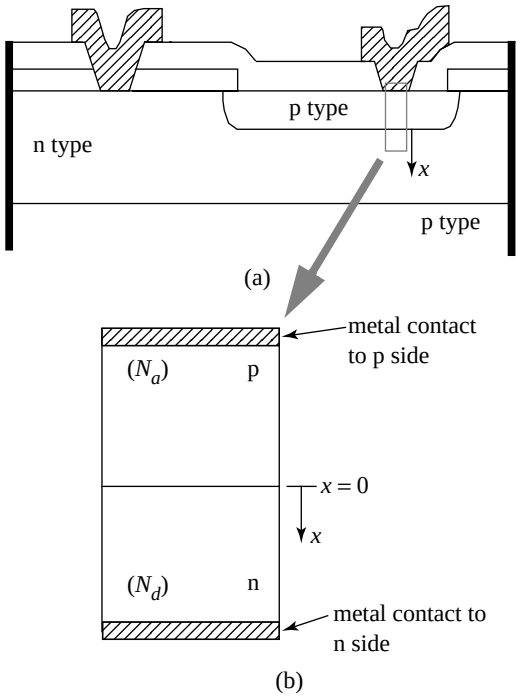


$$\phi_p = -\frac{kT}{q}\ln\left(\frac{N_A}{n_i}\right)$$

$$\phi_n = \frac{kT}{q}\ln\left(\frac{N_D}{n_i}\right)$$

pn Junctions

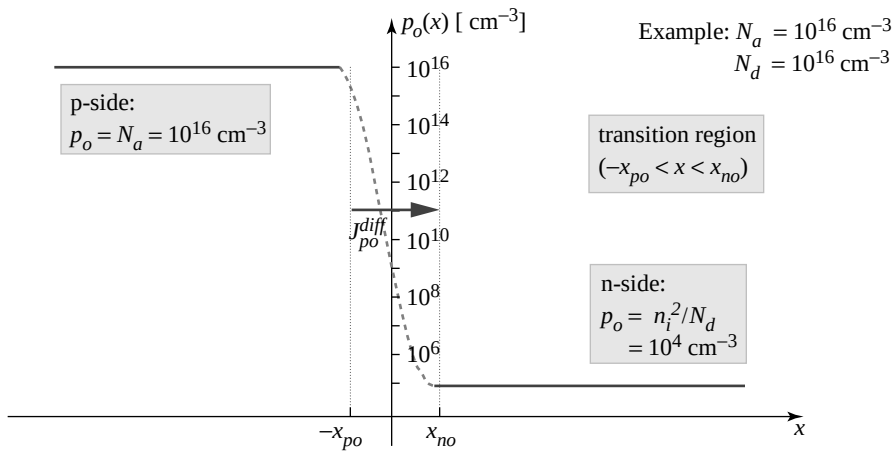
- ubiquitous IC structure -- pn junctions are everywhere!



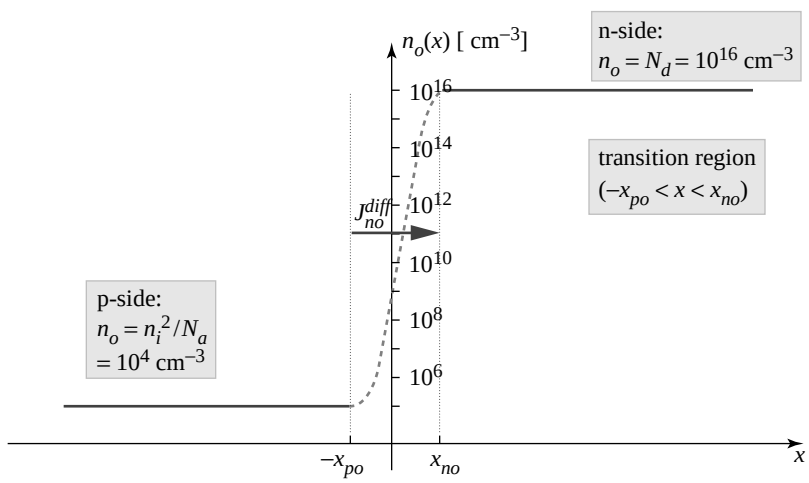
- thermal equilibrium: no hole current, no electron current ... no voltage applied between metal interconnects (could short them together)

Diffusion Currents in Thermal Equilibrium

- huge gradients in hole and electron concentration --> assume a transition region between - x_{po} and + x_{no}



(a)



(b)

note: we don't know how wide the transition region is (yet)

Drift and Diffusion in the Transition Region

- $J_{no} = 0$ and $J_{po} = 0$ due to equilibrium

  --> negative electric field in the transition region is needed ...

  where do + and - charges come from?

- Answer: the roll-off in electron concentration between $x = 0$ and x_{no} means that

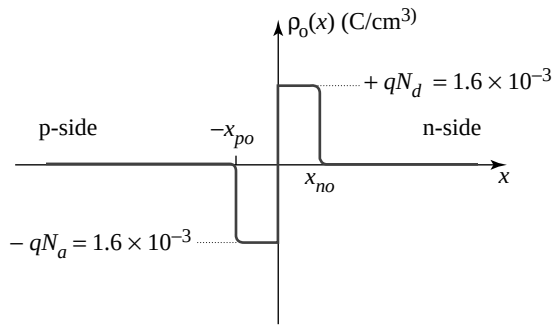
$$\rho_o(x) = q(-n_o(x) + N_d) > 0$$

  since $n_o(x) < N_d$ on the n-side of the transition region

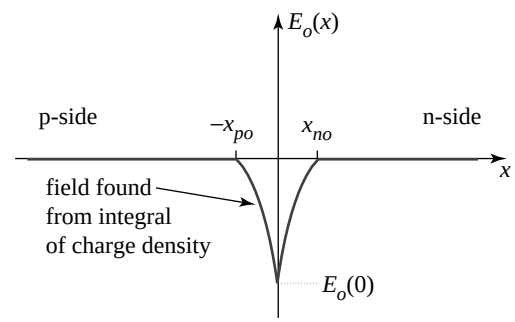
- On the p-side, the charge density is negative, since the hole concentration rolls off between $x = -x_{po}$ and $x = 0$.

Qualitative Electrostatics in Equilibrium

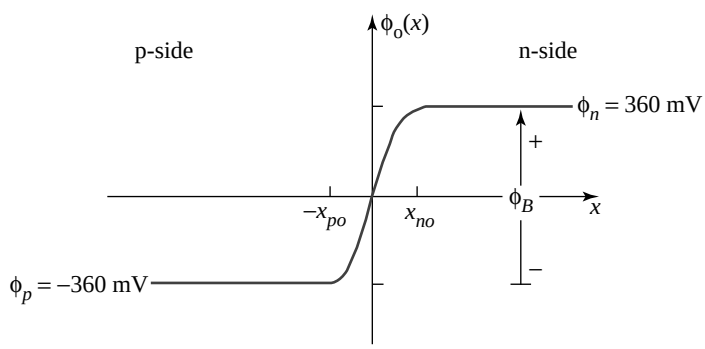
- From the charge density, we can find the electric field and the potential



(a)



(b)



(c)

Quantitative pn Junction in Thermal Equilibrium

The Depletion Approximation

- In the bulk regions far away from the junction, we can approximate

$$\rho = 0$$

- Near the junction, the charge density is non-zero. For example, on the n-side of the junction in the transition region, $0 < x < x_{no}$:

$$\rho = q\,(p_o + N_d - n_o - N_a) = q\,(N_d - n_o)$$

since there are no acceptors on this side ($N_a = 0$) and the hole concentration is negligible --> $p_o = 0$ (approx.)

The maximum positive value for charge density on the n-side is when there are *no electrons present in equilibrium* -- that is, when the silicon in the transition region is *depleted* of electrons.

For hand calculations, we will assume that

$$\rho = \rho_{max} = q\,N_d \quad (\,0 < x < x_{no}\,)$$

$$\rho = -\,q\,N_d \quad (\,-x_p < x < 0)$$

and proceed to find the width of the transition region, which we will rename the *depletion region*. The charge density is assumed to fall off *abruptly* from these values to zero in the bulk regions, where $x < -\,x_{po}$ and $x > x_{no}$

One more time ...

- Bulk silicon is *NEUTRAL*, to a good approximation

  >> region 1 is *bulk*:

$$\rho = q(N_d + p_o - N_a - n_o) \cong 0 \qquad \text{-->} \qquad p_o \cong N_a$$

  >> region 4 is *bulk*:

$$\rho = q(N_d + p_o - N_a - n_o) \cong 0 \qquad \text{-->} \qquad n_o \cong N_d$$

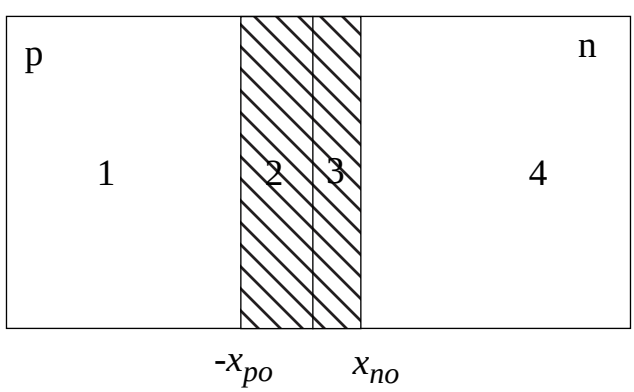
- Near the junction, the silicon is *DEPLETED* of mobile carriers:

  >> region 2 is *depleted*:

$$\rho = q(N_d + p_o - N_a - n_o) \cong -qN_a$$

  >> region 3 is *depleted*:

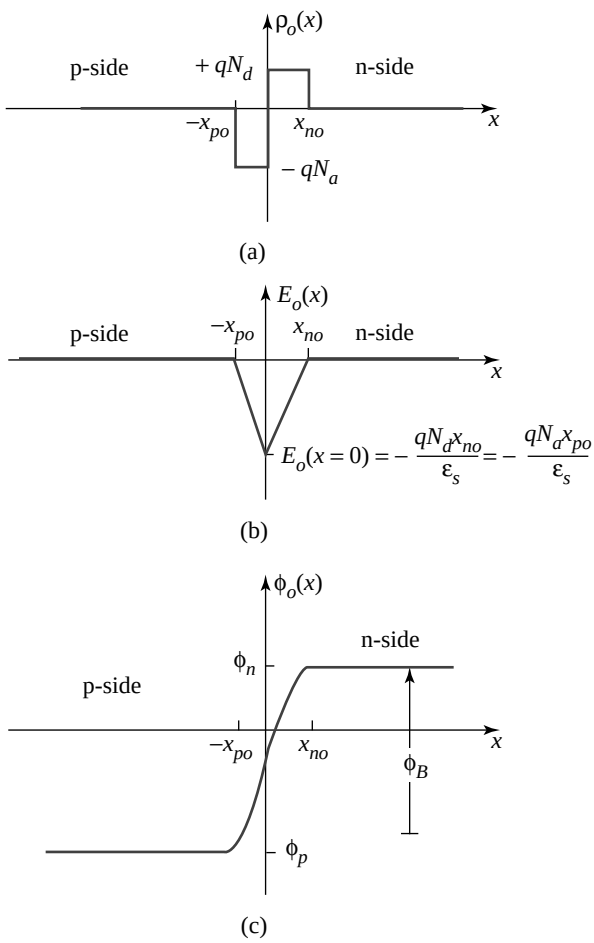
$$\rho = q(N_d + p_o - N_a - n_o) \cong qN_d$$



x_{no}, x_{po} are not known yet -- use boundary conditions to find them

pn Junction in Thermal Equilibrium:

Using the Depletion Approximation



- For detailed calculations, see Section 3.4. Analysis is straightforward, but involved. Use the fact that:

  > Charge in depletion region must sum to zero (why?)

  > Electrostatic potential is continuous

Depletion Widths in Thermal Equilibrium

$$x_{po} = \sqrt{\left(\frac{2\varepsilon_s \phi_B}{qN_a}\right)\left(\frac{N_d}{N_d + N_a}\right)}$$

$$x_{no} = \sqrt{\left(\frac{2\varepsilon_s \phi_B}{qN_d}\right)\left(\frac{N_a}{N_d + N_a}\right)}$$

$$X_{do} = x_{no} + x_{po} = \sqrt{\left(\frac{2\varepsilon_s \phi_B}{q}\right)\left(\frac{1}{N_a} + \frac{1}{N_d}\right)}$$

- Asymmetric junctions: i.e., $N_a >> N_d$ or $N_d >> N_a$.

  >> most of depletion width is on the side with the *lower* doping, since

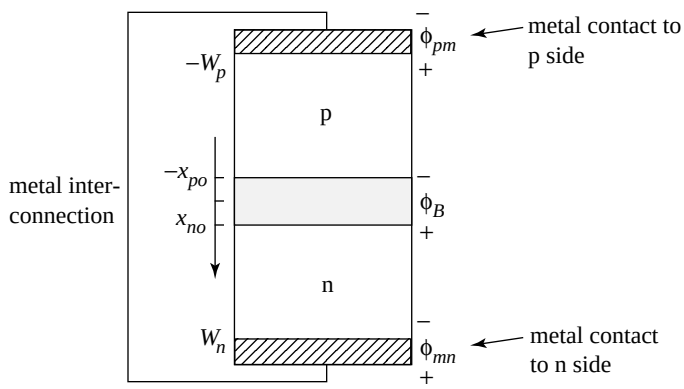
$$\frac{1}{N_a} + \frac{1}{N_d} \approx \frac{1}{N_d} \qquad (N_a \gg N_d)$$

$$\frac{1}{N_a} + \frac{1}{N_d} \approx \frac{1}{N_a} \qquad (N_d \gg N_a)$$

  >> most IC pn junctions are highly asymmetric

pn Junction under Reverse Bias

- First, we must understand the *complete* structure of the pn junction-- starting in thermal equilibrium:



- How can $V_D = 0$ and the built-in potential barrier be $\phi_B = 1$ V (approx.)?

  Answer: look at the complete circuit ... including the potential barriers at the p-type silicon-to-metal (ϕ_{pm}) and the metal-to-n-type silicon (ϕ_{mn}) junctions.

- Kirchhoff's Voltage Law:

$$0 = \phi_{pm} + \phi_B + \phi_{mn}$$

therefore, the built-in voltage is given by:

$$\phi_B = -\phi_{pm} - \phi_{mn}$$